Algebra Parts I & II

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Open Textbook Collaborative

The Open Textbook Collaborative (OTC) project is a statewide project managed by Middlesex College along with assistance from Brookdale Community College, Ocean County College , Passaic County Community College, and Rowan University .

The project engages a consortium of New Jersey community colleges, four year colleges and universities, and workforce partners to develop open educational resources (OER) in career and technical education STEM courses.

The courses align to career pathways in New Jersey's growth industries including health services, technology, energy, and global manufacturing and supply chain management as identified by the New Jersey Council of Community Colleges.

Beginning Algebra

OER

Workbook

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0.1 Integers

Learning Objectives: In this section, you will:

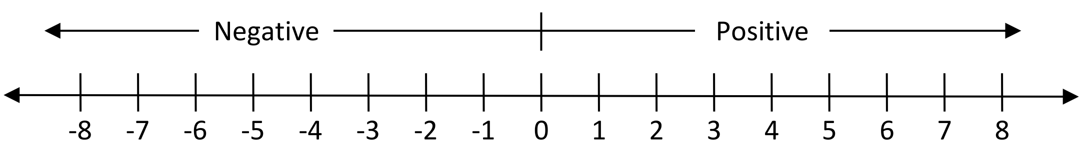
- Add, Subtract, Multiply and Divide Integers

Counting Numbers or **Natural Numbers**: 1,2,3,4…..
Whole Numbers: 0,1,2,3,4……..
Integers: Whole Numbers and their opposites, meaning they can be both positive and negative. Zero is also an integer. It is the only integer without a sign. …-2, -1, 0, 1, 2, 3, …

A common application of integers is temperature, which can be positive or negative, in both Fahrenheit and Celsius.

Example A) Use as integer: The temperature is 20 degrees below 0°.
If the temperature was 30 degrees *above* 0°, we'd just write 30°.
Since the temperature is 20 degrees *below* 0°, we write -20°.

We can visualize negative numbers using a number line. Values increase as you move to the right and decrease to the left.



Example B) Compare integers:
Write < or > to compare the numbers: a) 3 __ 5 b) -4 __ 3 c) -2 __ -5
a) On a number line, 3 is to the left of 5, so 3 < 5
b) On the number line, -4 is to the left of 3, so -4 < 3
c) On a number line, -2 is to the right of -5, so -2 > -5

Add and Subtract Integers

To add/subtract signed numbers of the **same sign** (both positive + + or both negative **- -**):
- **Add** the absolute values of the numbers
- **Keep the sign**

To add/subtract signed numbers of **opposite sign** (one positive, one negative: **+ -**):
- **Subtract** the smaller absolute value from the larger absolute value
- **Keep the sign of the larger** absolute value number

Rewrite subtraction as adding the opposite of the second number:
$a - b = a + (-b)$ and $a - (-b) = a + b$

Example C) Add: $-8 + (-5) =$
Since both numbers are negative, we add their absolute values: $8 + 5 = 13$
The result will be negative: $-8 + (-5) = -8 - 5 = -13$

Example D) Add: $-4 + 9$
The absolute values of the two numbers are 9 and 4. We subtract the smaller from the larger:
$9 - 4 = 5$
Since 9 had the larger absolute value and is positive, the result will be positive.

–4 + 9 = 5

Example E) Add: 5 + (–8)

The absolute values of the two numbers are 5 and 8. We subtract the smaller from the larger:
8 – 5 = 3
Since 8 had the larger absolute value and is negative, the result will be negative.
5 + (–8) = 5 – 8 = –3

Example F) Subtract: 10 – (–3)

We rewrite the subtraction as adding the opposite: 10 + 3 = 13

Example G) Simplify:	-3 +(-4) – 2 + 6 –(-5)	same signs: add, keep sign
work **left to right**:	-7 – 2 + 6 –(-5)	same signs: add, keep sign
	-9 + 6 –(-5)	opposite signs: subtract, keep sign of larger
	-3 + 5	opposite signs: subtract, keep sign of larger
	2	
OR **group all + and -** :	+6 +5 -3 -4 -2	add all + and add all –
	+11 – 9	opposite signs: subtract, keep sign of larger
	2	

<u>To multiply or divide two integers</u>

- If the two numbers have **different sign**, the result will be **negative**
- If the two numbers have the **same sign**, the result will be **positive**

Example H) Multiply: a) $-4 \cdot 3$ b) $5(-6)$ c) $-7(-4)$

a) The factors have different signs, so the result will be negative: $-4 \cdot 3 = -12$
b) The factors have different signs, so the result will be negative: $5(-6) = -30$
c) The factors have the same signs, so the result will be positive: $-7(-4) = 28$

Example I) Divide: a) $-40 \div 10$ b) $8 \div (-4)$ c) $\frac{-36}{-3}$

a) The numbers have different signs, so the result will be negative: $-40 \div 10 = -4$
b) The numbers have different signs, so the result will be negative: $8 \div (-4) = -2$
c) The numbers have the same signs, so the result will be positive: $\frac{-36}{-3} = 12$

===

Worksheet: 0.1 Integers

Write an integer for each situation:

1) I withdraw \$100 from my account 2) 20 feet above sea level 3) I lost 5 lbs

Write < or > to compare the numbers:

4) 207 __ 198 5) 23 __ -37 6) -2 __ -7 7) -152 __ -130

Add or Subtract:

8) –8 + 3 9) –1 - 13 10) 8 + (–6) 11) 120 + (–150)

12) 6-18+3 13) –10 – 8 –(-2) 14) -10 – (-4) 15) 26 – (-12)

16) The temperature was 29 degrees at 6 a.m. It went down 40 degrees by 12 noon. However, it increased by 15 degrees by 10 p.m. What was the temperature at 10 p.m.?

17) In Fargo it was -18°F, while in Tacoma it was 43°F. How much warmer was Tacoma?

18) Darrel's account was overdrawn by \$120, before he deposited \$450. What is his balance now?

Multiply or divide:

19) $-7 \cdot 4$ 20) $-5(-8)$ 21) $5(-3)$ 22) $-48 \div (-8)$

23) $\frac{-16}{-4}$ 24) $\frac{-10}{5}$

==

0.2 Fractions

Learning Objectives: In this section, you will:

- Reduce, add, subtract, multiply, and divide with fractions

Converting from mixed number to improper fraction

1. Multiply the whole number by the denominator of the fraction to determine how many pieces we have in the whole.
2. Add this to the numerator of the fraction
3. Use this sum as the numerator of the improper fraction. The denominator is the same.

Example A) Convert $5\frac{2}{7}$ to an improper fraction.

If we had 5 wholes, each divided into 7 pieces, that'd be $5 \cdot 7 = 35$ pieces.

Adding that to the additional 2 pieces gives 35+2 = 37 total pieces. The fraction would be $\frac{37}{7}$

Converting from improper fraction to mixed number

1. Divide: numerator ÷ denominator
2. The quotient is the whole part of the mixed number.
3. The remainder is the numerator of the mixed number. The denominator is the same.

Example B) Write $\frac{47}{6}$ as a mixed number. Dividing, 47÷6 = 7 remainder 5.

So, there are 7 wholes, and 5 remaining pieces, giving the mixed number $7\frac{5}{6}$

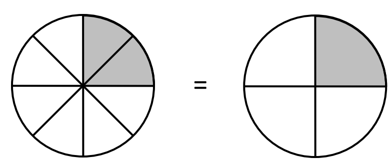
Equivalent fractions

To find equivalent fractions, **multiply or divide both the numerator and denominator** by the same number.

Example C) Write two fractions equivalent to $\frac{2}{8}$

By multiplying the top and bottom by 3, $\frac{2}{8} = \frac{2 \cdot 3}{8 \cdot 3} = \frac{6}{24}$

By dividing the top and bottom by 2, $\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$



Multiply fractions

To multiply two fractions, you multiply the numerators, and multiply the denominators: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

Example D) Multiply and simplify $\frac{2}{3} \cdot \frac{5}{8}$

$\frac{2}{3} \cdot \frac{5}{8} = \frac{2 \cdot 5}{3 \cdot 8} = \frac{10}{24}$, which we can simplify to $\frac{5}{12}$ by dividing numerator and denominator by 2.

Alternatively, we could have noticed that in $\frac{2 \cdot 5}{3 \cdot 8}$, the 2 and 8 have a common factor of 2, so we can divide the numerator and denominator by 2, often called "cancelling" the common factor: $\frac{2 \cdot 5 \div 2}{3 \cdot 8 \div 2} = \frac{1 \cdot 5}{3 \cdot 4} = \frac{5}{12}$

To multiply with mixed numbers, it is easiest to first convert the mixed numbers to improper fractions.

Example E) Multiply and simplify $3\frac{1}{3} \cdot 4\frac{4}{5}$

Converting these to improper fractions first, $3\frac{1}{3} = \frac{10}{3}$ and $4\frac{4}{5} = \frac{24}{5}$, so $3\frac{1}{3} \cdot 4\frac{4}{5} = \frac{10}{3} \cdot \frac{24}{5}$

$\frac{10}{3} \cdot \frac{24}{5} = \frac{10 \cdot 24}{3 \cdot 5}$. Since 5 and 10 have a common factor of 5, we can cancel that factor: $\frac{2 \cdot 24}{3 \cdot 1}$

Since 3 and 24 have a common factor of 3, we can cancel that factor: $\frac{2\cdot 8}{1\cdot 1}=\frac{16}{1}=16$

Divide fractions

To divide two fractions, **multiply the first number by that reciprocal of the second number** (reciprocal: convert number to upside down form).

Example F) Divide and simplify $\frac{5}{8}\div\frac{5}{6}$

We find the reciprocal of $\frac{5}{6}$ and change this into a multiplication problem:

$$\frac{5}{8}\cdot\frac{6}{5}=\frac{5\cdot 6}{8\cdot 5}=\frac{1\cdot 3}{4\cdot 1}=\frac{3}{4}$$

Example G) Divide and simplify $5\frac{1}{2}\div 1\frac{1}{3}$

Rewriting the mixed numbers first as improper fractions, $\frac{11}{2}\div\frac{4}{3}$

We find the reciprocal of $\frac{4}{3}$ and change this into a multiplication problem

$$\frac{11}{2}\cdot\frac{4}{3}=\frac{11\cdot 4}{2\cdot 3}=\frac{11\cdot 2}{1\cdot 3}=\frac{22}{3}=7\frac{1}{3}$$

Add and subtract fractions with like denominators

We can only add or subtract fractions with **like** denominators. To do this, we add or subtract the numerators. The denominator remains the same: $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ and $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$

Example H) Add and simplify $\frac{7}{9}+\frac{5}{9}$

$$\frac{7}{9}+\frac{5}{9}=\frac{7+5}{9}=\frac{12}{9}=1\frac{3}{9}=1\frac{1}{3}$$

Add and subtract with unlike denominators

1. Find common denominators
2. Change fractions to equivalent forms having common denominators
3. Add or subtract the numerators. The denominator remains the same

Example I) Add and simplify $\frac{1}{4}+\frac{1}{2}$

Since these don't have the same denominator, we identify the least common multiple of the two denominators, 4, and give both fractions that denominator. Then we add and simplify.

$$\frac{1}{4}+\frac{1}{2}=\frac{1}{4}+\frac{2}{4}=\frac{1+2}{4}=\frac{3}{4}$$

Example J) Subtract and simplify $\frac{5}{8}-\frac{7}{12}$

The least common multiple of 8 and 12 is 24. We give both fractions this denominator and subtract. $\frac{5}{8}-\frac{7}{12}=\frac{15}{24}-\frac{14}{24}=\frac{1}{24}$

Example K) Add and simplify $2\frac{2}{3}+5\frac{3}{4}$

Rewriting the fractional parts with a common denominator of 12: $2\frac{8}{12}+5\frac{9}{12}$

Adding the whole parts $2+5=7$. Adding the fractional parts, $\frac{8}{12}+\frac{9}{12}=\frac{17}{12}=1\frac{5}{12}$.

Now we combine these: $7+1\frac{5}{12}=8\frac{5}{12}$

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<u>Worksheet: 0.2 Fractions</u>

Convert each mixed number to an improper fraction 1) $4\frac{3}{4}$ 2) $1\frac{7}{16}$

Convert each improper fraction to a mixed number 3) $\frac{35}{2}$ 4) $\frac{15}{6}$

Simplify to lowest terms

5) $\frac{3}{6}$ 6) $\frac{10}{12}$ 7) $\frac{150}{130}$ 8) $\frac{24}{18}$

Multiply and simplify

9) $\frac{2}{5}\cdot\frac{3}{4}$ 10) $12\cdot\frac{2}{3}$ 11) $\frac{3}{10}\cdot\frac{2}{5}\cdot(-\frac{5}{9})$ 12) $8\frac{1}{6}\cdot 4\frac{2}{7}$

13) One dose of eyedrops is $\frac{1}{8}$ ounce. How many ounces are required for 40 doses?

Divide and simplify

14) $\frac{3}{5}\div\frac{1}{4}$ 15) $18\div\left(-\frac{2}{3}\right)$ 16) $3\frac{1}{4}\div\frac{1}{6}$ 17) $2\frac{2}{5}\div 4\frac{1}{3}$

18) One dose of eyedrops is $\frac{1}{8}$ ounce. How many doses can be administered from 4 ounces?

Add or Subtract and simplify

19) $\frac{3}{10}+\frac{5}{10}$ 20) $\frac{2}{7}-\frac{4}{7}$ 21) $\frac{2}{5}+\frac{1}{3}$ 22) $\frac{3}{8}+\frac{1}{6}$

23) $\frac{9}{14}+\left(-\frac{20}{21}\right)$ 24) $-3\frac{1}{4}-2\frac{1}{2}$ 25) $8\frac{2}{3}+6\frac{3}{4}$ 26) $\frac{4}{5}-\left(-\frac{7}{10}\right)$

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0.3 Order of Operations

Learning Objectives: In this section, you will:

- Evaluate expressions using the order of operations

Order of Operations

When we combine multiple operations, we need to agree on an order to follow, so that if two people calculate $2 + 3 \cdot 4$they will get the same answer. To remember the order, some people use the mnemonic PEMDAS:

P:	**P**arentheses
E:	**E**xponents and roots
MD:	**M**ultiplication and **D**ivision
AS:	**A**ddition and **S**ubtraction

IMPORTANT!! Notice that multiplication and division have the SAME precedence, as do addition and subtraction. When you have multiple operations of the same level, you work left to right.

Order of Operations

Step 1. Do anything that is inside parentheses
Step 2. Solve anything that contains an exponent (a power – 5^3 – the 3 is the exponent and it means the base number is to be multiplied by itself that number of times, so $5^3 = 5(5)(5) = 125$)
Step 3. Solve any multiplication or division within the problem, moving from left to right
Step 4. Solve any addition or subtraction within the problem, moving from left to right

Example A) Simplify: $-3 - 5\left(3^2 + 6 \div (-2)\right)$

We begin with the inside of the parenthesis, with the exponent:	$-3 - 5\left(9 + 32 \div (-2)\right)$
Still inside the parenthesis, we do the division:	$-3 - 5\left(9 + (-16)\right)$
Inside the parenthesis, we add $9 + (-16) = 9 - 16 = -7$	$-3 - 5(-7)$
Now multiply -5(-7) = 35	$-3 + 35$
Add	32

Example B)

P	$-2(12 - 8) + (-3)^3 + 4 \bullet (-6)$
E	$-2(4) + (-3)^3 + 4 \bullet (-6)$
MD left to right	$-2(4) + (-27) + 4 \bullet -6$
AS left to right	$-8 - 27 - 24$
AS left to right	$-35 - 24$
	-59

Example C)

P	$-3 + 4(2 - 6)^2 \div (-2)$
E	$-3 + 4(-4)^2 \div (-2)$
MD left to right	$-3 + 4(16) \div (-2)$
MD	$-3 + 64 \div (-2)$
AS	$-3 - 32$
	-35

If the operations to be performed are in fractional form, solve the numerator first, then the denominator, then reduce.

Example D)

$$\frac{2^4-(-8)\cdot 3}{15\div 5-1}$$ Exponent in the numerator, divide in denominator

$$\frac{16-(-8)\cdot 3}{3-1}$$ Multiply in the numerator, subtract in denominator

$$\frac{16-(-24)}{2}$$ Add the opposite to simplify numerator, denominator is done.

$$\frac{40}{2}$$ Reduce, divide

$$20$$ Our Solution

Example E)

P $\frac{2}{5}\left(\frac{2}{3}-\left(\frac{1}{2}\right)^2\right)$ Start with ( ), inside ( ): exponents first: $\left(\frac{1}{2}\right)^2=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$

P $\frac{2}{5}\left(\frac{2}{3}-\frac{1}{4}\right)$ Continue ( ): $\frac{2}{3}-\frac{1}{4}=(find\ common\ deominator)=\frac{(8-3)}{12}=\frac{5}{12}$

M $\frac{2}{5}\cdot\frac{5}{12}=(cross\ cancel)=\frac{1}{1}\cdot\frac{1}{6}=\frac{1}{6}$ Multiply

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<u>Worksheet: 0.3 Order of Operations</u>

Simplify:

1) $4-(-5)\cdot 6$

2) $(-3)^2-4^2$

3) 18 - (-12 - 3)

4) $-19 + (7+4)^2$

5) $20 - 4(3^2 - 6)$

6) $-3 + 2(-6 \div 3)^2$

7) $\frac{3-4(5-7)}{1+6\div 3}$

8) $\frac{1}{4}-\frac{3}{4}\cdot\frac{1}{6}$

9) $-6(12 - 15) + 2^3$

10) $\frac{4(-6)+8-(-2)}{15-7+2}$

11) $\frac{1}{3}(5+2)$

12) $4\left(\frac{7}{8}-\frac{1}{4}\right)$

13) $5\cdot\left(\frac{1}{2}\right)^2$

14) $\left(\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^3$

15) $\frac{1}{5}(7)-\frac{2}{3}\left(\frac{1}{2}\right)$

16) Jean's three pea plants measure 6½, 5¼, and 4 inches tall. Find the mean (average) height.
(mean = add up all the numbers, then divide by how many numbers there are)

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<u>0.4 Properties of Algebra (Simplify, Evaluate, Translate Expressions)</u>

Learning Objectives: In this section, you will:

- Simplify expressions
- Evaluate expressions
- Combine like terms
- Translate algebraic expressions

Vocabulary:

- **Algebraic Expression:** An expression that contains at least one variable.
  - ***Example:*** $x + 8$ or $4(m - b)$
- **Terms:** All the parts of expressions or equations. Term is a number, a variable, or a product or quotient of numbers and variables raised to powers.
  - ***Example:*** $2x^2+3x-18$ terms are $2x^2$, $3x$, and -18
- **Variable:** A symbol used to represent a quantity that can change. This is usually a letter.
  - ***Example:*** In $3x + 8$, x is the variable.
- **Constant:** A value that does not change.
  - ***Example:*** In $3x + 8$, 8 is the constant.
- **Coefficient:** The number that is multiplied by the variable in an algebraic expression.
  - ***Example:*** In $3x + 8$, 3 is the coefficient.
  - ***Example:*** $2x+3$, variable: x, coefficient: 2, constant: 3
- **Like terms**: terms with exactly the same variables that have the same exponents on the variable are like terms.
  - ***Example:*** of like terms would be: $3xy$ & $-7xy$ OR $8a^2b$ & $-2a^2b$

Evaluate algebraic expressions: Replace the variables with their numerical values and follow order of operations.

Example A) Evaluate $p(q + 6)$ when $p = -3$ and $q = 5$

Replace p with -3 and q with -5:	$(-3)(5 + 6)$
Evaluate parenthesis, add:	$(-3)(11)$
Multiply:	-33 Our solution

Combine like terms

If we have like terms we are allowed to add (or subtract) the numbers in front of the variables (called coefficients), then keep the variables the same.

Example B) Simplify: $8x^2 - 3x + 7 - 2x^2 + 4x - 3$

$8x^2 - 3x + 7 - 2x^2 + 4x - 3$	Combine like terms $8x^2 - 2x^2$ and $-3x + 4x$ and $7 - 3$
$6x^2 + x + 4$	Our solution

Distributive Property: multiply a sum or difference, multiply each term by 'a'

$a(b + c) = ab + ac$

Example C) Simplify: $4(2x - 7)$

$4(2x - 7)$	Multiply each term by 4
$8x - 28$	Our Solution

Example D) Simplify: - *(4x – 5y + 6)*

$-(4x - 5y + 6)$	Negative can be thought of as -1
$-1(4x - 5y + 6)$	Multiply each term by -1
$-4x + 5y - 6$	Our Solution

Example E) Simplify*: 2(5x -8) -6(4x +3)*

$2(5x - 8) - 6(4x + 3)$	Distribute 2 into first parenthesis and -6 into second
$10x - 16 - 24x - 18$	Combine like terms $10x - 24x$ and $-16 - 18$
$-14x - 34$	Our Solution

<u>Translate algebraic expressions</u>

Operation	+	-	×	÷
Algebraic Expression	x + 28	k - 12	8 • w (8)(w) 8w 8(w)	n ÷ 3 $\frac{n}{3}$
Words or Phases	• 28 **added to** x • x **plus** 28 • The **sum of** x and 28 • 28 **more than** x	• 12 **subtracted from** k **<u>(reverse order!)</u>** • 12 **less than** k **<u>(reverse order!)</u>** • **Take away** 12 from k **<u>(reverse order!)</u>** • k **minus** 12 • the **difference** of k and 12	• 8 **times** w • w **multiplied** by 8 • The **product** of 8 and w • 8 groups of w	• n **divided by** 3 • The **quotient** of n and 3

Example F) Translate: the product of 8 and 5 less than a number

- product: multiply, place multiplication where 'and' is,
  - '8' is before multiplication, '5 less than a number' is after
- 5 less than a number:
  - a number is any variable: 'x'
  - '5 less than a number' subtraction, reverse order: x-5
- Translation: the product of 8 and 5 less than a number

  8(x-5)

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Worksheet: 0.4 Properties of Algebra (Simplify Expressions)

Evaluate each expression if $\boldsymbol{x = -2}$, $\boldsymbol{y = 4}$, and $\boldsymbol{z = 6}$.

1. $x^3 + 10y$

2. $\frac{22}{x} + 16$

3. $yz \div x^2$

Evaluate each expression if $\boldsymbol{r = -4}$, $\boldsymbol{s = 6}$, $\boldsymbol{t = -3}$, and $\boldsymbol{u = 12}$.

4. $2r + st^2 - u$

5. $u - (r - 3s)$

6. $\dfrac{2r(s-t)}{tu-s}$

Combine like terms:

7) − 7x − 2x 8) x − 10 − 6x + 1 9) m − 2m 10) 9n − 1+ n + 4

Distribute:

11) 3(8v +9) 12) − (−5+9a) 13) -10(1 +2x) 14) − 2(n +1)

Simplify:

15) 12p - (2p -1) 16) 9(b +10) +5b 17) 4v -7(1-8v) 18) -3(y-4x) – (x-3y)

Translate: write an algebraic expression to the given verbal expression.

19. eight less than a number
20. a number increased by seven
21. the quotient of *m* and *n*
22. a number squared
23. the sum of 3 times *a* and *b*
24. three times the sum of *a* and *b*
25. seven more than the cube of a number
26. one-half the product of *x* and *y*
27. the product of twice *a* and *b*
28. twice the product of *a* and *b*
29. two less than five times a number
30. twice a number increased by three times the number
31. the cube of *a* plus *b*
32. the cube of the sum of *a* and *b*

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1.1 Solving Linear Equations-One Step Equations

Learning Objectives

In this section, you will:

- Solve one step linear equations by balancing using inverse operations.

An <u>equation</u> is a statement asserting that two algebraic expressions are equal.

<u>Solving equations</u> means to get the variable by itself (isolate).
→ Note: The answer should look like (variable) = (some number), where the variable is never negative

<u>Solve using addition and subtraction.</u>
Example A) Solve: r + 16 = -7

$r + 16 = -7$ Get the variable by itself. Right now 16 is being added to it.
$-16 \quad -16$ Undo the addition by subtracting 16 from both sides.
$r = -23$ Answer.

Subtraction Property of Equality Subtracting the same value from both sides of the equation.

Note: What ever you do to one side, you MUST do to the other side (keep it balanced).

When solving equations, eliminate double signs.
→As a general rule, replace "+ (-)" with "–" and "– (-)" with "+".

Example B) Solve: y + (-3) = 8

$y + (-3) = 8$
$y - 3 = 8$ Undo the subtraction by adding 3 to both sides.
$+3 \quad +3$
$y = 11$ Answer.

Addition Property of Equality
Adding the same value from both sides of the equation.

<u>Solve using multiplication and division.</u>
Example C) Solve: -5t = 60

$-5t = 60$ Get the variable by itself. Right now -5 is being multiplied to it.
$\frac{-5t}{-5} = \frac{60}{5}$ Undo the multiplication by dividing both sides by -5.
$t = -12$ Answer.

Division Property of Equality

Dividing the same value from both sides of the equation.

Remember: What ever you do to one side, you MUST do to the other side (keep it balanced).

Example D) Solve: $\frac{x}{4} = -12$

$\frac{x}{4} = -12$ Since 4 is dividing x, multiply both sides by 4 to clear the fraction. $(4)\frac{x}{4} = -12(4)$

$\frac{4x}{4} = -48$ The fours will cancel each other out. $\frac{4x}{4}$ simplifies to 1x

$x = -48$

Multiplication Property of Equality
Multiplying the same value from both sides of the equation.

Example E) Solve: $\frac{2}{3}x = 18$

$\frac{2}{3}x = 18$ To get rid of multiplying a fraction, multiply by the reciprocal.

$\left(\frac{3}{2}\right)\frac{2}{3}x = 18\left(\frac{3}{2}\right)$ Multiply straight across.

$x = \frac{54}{2}$

$x = 27$

Check solution: verify that a given value is a solution to an equation. The two sides must balance.

Example F) Verify that *x=7* is the solution to the algebraic equation *x - 5= 2*.
We replace *x* with 7 in the equation.

x-5 = 2

7 -5 $=^?$ 2

2 = 2

So, 7 is the solution to the x-5=2

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Worksheet: 1.1 Solving Linear Equations-One Step Equations.

1) $v + 9 = 16$

2) $14 = b + 3$

3) $x - 11 = -16$

4) $-14 = x - 18$

5) $30 = a + 20$

6) $-1 + k = 5$

7) $x - 7 = -26$

8) $-13 + p = -19$

9) $13 = n - 5$

10) $22 = 16 + m$

11) $340 = -17x$

12) $4r = -28$

13) $-9 = \frac{n}{12}$

14) $\frac{5}{9} = \frac{b}{9}$

15) $20v = -160$

16) $-20x = -80$

<u>1.2 Linear Equations- Two Steps Equations</u>

Learning Objectives: In this section, you will:

- Solve a two-step equation using addition, subtraction, multiplication, and division.
- Solve a two-step equation by combining like terms.
- Solve real-world problems using two-step equations.

<u>TWO-STEP EQUATIONS:</u> Work Backwards

1) Find the **<u>term</u>** with the **<u>VARIABLE, you want to isolate that term and then the variable.</u>**

2) Identify **<u>all operations that is happening to the variable</u>.**

3) In **<u>REVERSE ORDER of PEMDAS</u>** (SADMEP), cancel operations.

4) Use the **<u>same number</u>** and **<u>opposite operations</u> to both sides of the equation.**

Example A) Solve for x: $13 = 5 + 2x$

Step 1: Find the term with the variable	$13 = 5 + \underline{\mathbf{2x}}$	
Step 2: What is happening to that term?	$13 = \underline{\mathbf{5+}}\, 2x$	5 is being added to $2x$
Step 3: Do the opposite operation to both sides of the equation.	$13 = 5 + 2x$ -5 -5	Subtraction is the opposite operation of addition, so subtract 5 from both sides.
Step 4: Find the variable.	$8 = 2\underline{\mathbf{x}}$	
Step 5: What is happening to the variable?	$8 = \underline{\mathbf{2}}x$	x is being multiplied by 2 (remember that any time there is a number followed by a variable in algebra, it means multiply).
Step 6: Do the opposite to both sides of the equation.	$\frac{8}{2} = \frac{2x}{2}$	Division is the opposite operation of multiplication, so divide both sides by 2.
	$4 = x$	See that $x = 4$.
Check:	$13 = 5 + 2(4)$ $13 = 5+8$	Substitute the answer back into the original equation. Since 13 is 13, our answer is correct.

Example B) Solve: $5x + 7 = 7$

$$\begin{array}{rcll} 5x+7= & 7 & & \text{Start by focusing on the plus 7} \\ \underline{-\mathbf{7}} & \underline{-\mathbf{7}} & & \text{Subtract 7 from both sides} \\ 5x & = 0 & & \text{Now focus on the multiplication by 5} \\ \overline{\mathbf{5}} & \overline{\mathbf{5}} & & \text{Divide both sides by 5} \\ x= & 0 & & \text{Our Solution!} \end{array}$$

Example C) Solve: $4 - 2x = 10$

$$\begin{array}{rcll} 4-2x= & 10 & & \text{Start by focusing on the positive 4} \\ \underline{-\mathbf{4}} & \underline{-\mathbf{4}} & & \text{Subtract 4 from both sides} \\ -2x & =6 & & \text{Negative (subtraction) stays on the } 2x \\ \overline{-\mathbf{2}} & \overline{-\mathbf{2}} & & \text{Divide by } -2 \\ x= & -3 & & \text{Our Solution!} \end{array}$$

Example D) **Real world problem, solve:** An emergency plumber charges $65as a call-out fee plus an additional $75 per hour. He arrives at a house at 9:30 AM and works to repair a water tank. If the total repair bill is $196.25, at what time was the repair completed?

- We collect the information from the text and convert it to an equation.
- Unknown time taken in hours this will be our 'x'
- The bill is made up of two parts: a call out fee + a per-hour fee.
  - $65 as a call-out fee 65: independent of x
  - Plus an additional $75 per hour +75x
  - Total Bill = 65+75x
- The total on the bill was $196.25. So, our final equation is: 196.25 = 65+75x

196.25 = 65+75x
− 65 −65 To isolate x, first subtract 65 from both sides
131.25 = 75x Divide both sides by 75

$\frac{131.25}{75} = x$

x = 1.75 The time taken was one- and three-quarter hours (1 hr. 45 min)

Solution: The repair job was completed at 9:30 + 1:45 = 11:15AM

==

Worksheet: 1.2 Solving Linear Equations-Two Step Equations.

1) $5+\frac{n}{4}=4$

2) $-2=-2m+12$

3) $102=-7r+4$

4) $27=21-3x$

5) $-8n+3=-77$

6) $-4-b=8$

7) $0=-6v$

8) $-2+\frac{x}{2}=4$

9) $-8=\frac{x}{5}-6$

10) $-5=\frac{a}{4}-1$

11) The product of negative 4 and y increased by 11 is equivalent to -5.

12) Eight more than five times a number is negative 62.

13) You bought a magazine for $5 and four erasers. You spent a total of $25. How much did each eraser cost?

14)
Jade is stranded downtown with only $10 to get home. Taxis cost $0.75 per mile, but there is an additional $2.35 hire charge. Write a formula and use it to calculate how many miles she can travel with her money. Determine how many miles she can ride.

15)
Jasmins Dad is planning a surprise birthday party for her. He will hire a bouncy castle, and will provide party food for all the guests. The bouncy castle costs $150 dollars for the afternoon, and the food will cost $3.00 per person. Andrew, Jasmins Dad, has a budget of $300 . Write an equation to help him determine the maximum number of guests he can invite.

==

1.3 General Linear Equations- Multi Steps Equations

Learning Objectives: In this section, you will:

- Solve a multi-step equation by combining like terms
- Solve a multi-step equation by the distributive property
- Solve real-world problems using multi-step equations

How to solve a linear equation and find the value of the variable (x):
1) Use Distributive Property to remove any parentheses, if necessary. 2) Combine like terms on each side, if necessary. 3) Move all variables (x-terms) to one side of = sign by adding/subtracting. 4) Move all number-terms (constants) to other side of = sign by adding/subtracting. 5) Combine like terms on each side. 6) Divide both sides by coefficient of x-term to find 'x'. 7) Check the solution.

Example A) Solve: $-3x + 9 = 6x - 27$

$$\begin{array}{rl}
-3x + 9 = 6x - 27 & \text{Notice the variable on both sides, } -3x \text{ is smaller} \\
\underline{+\mathbf{3x} \quad +\mathbf{3x}} & \text{Add } 3x \text{ to both sides} \\
9 = 9x - 27 & \text{Focus on the subtraction by 27} \\
\underline{+\mathbf{27} \quad +\mathbf{27}} & \text{Add 27 to both sides} \\
36 = 9x & \text{Focus on the mutiplication by 9} \\
\overline{\mathbf{9}} \quad \overline{\mathbf{9}} & \text{Divide both sides by 9} \\
4 = x & \text{Our Solution}
\end{array}$$

Check: $-3(4) + 9 = 6(4) - 27$

$-12 + 9 = 24 - 27$

$-3 = -3$ True, so x = 4 is the solution.

Example B) Solve: $4(2x - 6) = 16$

$$\begin{array}{rl}
4(2x - 6) = 16 & \text{Distribute 4 through parenthesis} \\
8x - 24 = 16 & \text{Focus on the subtraction first} \\
\underline{+\mathbf{24} + \mathbf{24}} & \text{Add 24 to both sides} \\
8x = 40 & \text{Now focus on the multiply by 8} \\
\overline{\mathbf{8}} \quad \overline{\mathbf{8}} & \text{Divide both sides by 8} \\
x = 5 & \text{Our Solution!}
\end{array}$$

Check: $4(2(5) - 6) = 16$

$4(10 - 6) = 16$

$4(4) = 16$ True, so x = 5 is the solution.

An equation that is true for one or more values of the variable (like the ones above) and false for all other values of the variable is a **conditional equation**.

Example C) Solve: $3(2x - 5) = 6x - 15$

$$3(2x-5) = 6x - 15 \quad \text{Distribute 3 through parenthesis}$$
$$6x - 15 = 6x - 15 \quad \text{Notice the variable on both sides}$$
$$\underline{-\mathbf{6x} \qquad -\mathbf{6x}} \quad \text{Subtract } 6x \text{ from both sides}$$
$$-15 = -15 \quad \text{Variable is gone! True!}$$

When you solve an equation and you end with a **True statement**, the solution set will be: **Many Solutions or All Real Numbers.** This type of equation is called an **Identity.**

Example D) Solve: $2(3x - 5) - 4x = 2x + 7$

$$2(3x-5) - 4x = 2x + 7 \quad \text{Distribute 2 through parenthesis}$$
$$6x - 10 - 4x = 2x + 7 \quad \text{Combine like terms } 6x - 4x$$
$$2x - 10 = 2x + 7 \quad \text{Notice the variable is on both sides}$$
$$\underline{-\mathbf{2x} \qquad -\mathbf{2x}} \quad \text{Subtract } 2x \text{ from both sides}$$
$$-10 \neq 7 \quad \text{Variable is gone! False!}$$

When you solve an equation and you end with a **False statement**, the solution set will be: **No Solutions.** This type of equation is called a **Contradiction.**

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	**True** for any value of the variable	All real numbers
Contradiction	**False** for all values of the variable	No solution

==

Worksheet: 1.3 General Linear Equations

Solve each equation. Then state whether the equation is a conditional equation, an identity, or a contradiction.

1) $20 - 7b = -12b + 30$

2) $6x + 12 - 11x = -7 + 9x + 15$

3) $9(2m - 3) - 8 = 4m + 7$

4) $-2(8y - 4) = 8(1 - y)$

5) $-2 - 5(2 - 4m) = 33 + 5m$

6) $12 + 2(5 - 3y) = -9(y - 1) - 2$

7) $4(p - 4) - (p + 7) = 5(p - 3)$

8) $15y + 32 = 2(10y - 7) - 5y + 46$

9) $11(8c + 5) - 8c = 2(40c + 25) + 5$

10) $23x + 19 = 3(5x - 9) + 8x + 6$

==

1.4 Solving with Fractions

Learning Objectives: In this section you will:

- Solve linear equations with rational coefficients by multiplying by the least common denominator to clear the fractions

Solve Equations with Fraction Coefficients

Solve equations with fractions: **multiply each term by Least Common Denominator**. This step will change each coefficient to whole number (the equations stay equivalent to each other). This process is called ***clearing* the equation of fractions**.

Example A) Solve: $\frac{1}{12}x + \frac{5}{6} = \frac{3}{4}$

Solution

Step 1. Find the least common denominator of *all* the fractions and decimals in the equation.	What is the LCD of $\frac{1}{12}$, $\frac{5}{6}$, and $\frac{3}{4}$?	$\frac{1}{12}x + \frac{5}{6} = \frac{3}{4}$ LCD = 12
Step 2. Multiply both sides of the equation by that LCD. This clears the fractions and decimals.	Multiply both sides of the equation by the LCD, 12.	$12\left(\frac{1}{12}x + \frac{5}{6}\right) = 12\left(\frac{3}{4}\right)$
	Use the Distributive Property.	$12 \cdot \frac{1}{12}x + 12 \cdot \frac{5}{6} = 12 \cdot \frac{3}{4}$
	Simplify—and notice, no more fractions!	$x + 10 = 9$
Step 3. Solve using the General Strategy for Solving Linear Equations.	To isolate the variable term, subtract 10.	$x + 10 - 10 = 9 - 10$
	Simplify.	$x = -1$
		Check: $\frac{1}{12}x + \frac{5}{6} = \frac{3}{4}$ $\frac{1}{12}(-1) + \frac{5}{6} \stackrel{?}{=} \frac{3}{4}$ $-\frac{1}{12} + \frac{5}{6} \stackrel{?}{=} \frac{3}{4}$ $-\frac{1}{12} + \frac{10}{12} \stackrel{?}{=} \frac{9}{12}$ $\frac{9}{12} = \frac{9}{12}$ ✓

Example B) Solve:

$$\begin{aligned}
\frac{3}{4}x-\frac{7}{2}&=\frac{5}{6} && \text{LCD} = 12\text{, multiply each term by } 12\\
\frac{(\mathbf{12})3}{4}x-\frac{(\mathbf{12})7}{2}&=\frac{(\mathbf{12})5}{6} && \text{Reduce each } 12 \text{ with denominators}\\
(\mathbf{3})3x-(\mathbf{6})7&=(\mathbf{2})5 && \text{Multiply out each term}\\
9x-42&=10 && \text{Focus on subtraction by } 42\\
\underline{\mathbf{+42}\ \ \mathbf{+42}}& && \text{Add } 42 \text{ to both sides}\\
9x&=52 && \text{Focus on multiplication by } 9\\
\overline{\mathbf{9}}\quad\overline{\mathbf{9}}& && \text{Divide both sides by } 9\\
x&=\frac{52}{9} && \text{Our Solution}
\end{aligned}$$

In the next example, notice that the 2 is not a fraction in the original equation, but to solve it we put the 2 over 1 to make it a fraction.

Example C) Solve:

$$\begin{aligned}
\frac{2}{3}x-2&=\frac{3}{2}x+\frac{1}{6} && \text{LCD} = 6\text{, multiply each term by } 6\\
\frac{(\mathbf{6})2}{3}x-\frac{(\mathbf{6})2}{1}&=\frac{(\mathbf{6})3}{2}x+\frac{(\mathbf{6})1}{6} && \text{Reduce 6 with each denominator}\\
(\mathbf{2})2x-(\mathbf{6})2&=(\mathbf{3})3x+(\mathbf{1})1 && \text{Multiply out each term}\\
4x-12&=9x+1 && \text{Notice variable on both sides}\\
\underline{\mathbf{-4x}\qquad\mathbf{-4x}}& && \text{Subtract } 4x \text{ from both sides}\\
-12&=5x+1 && \text{Focus on addition of } 1\\
\underline{\mathbf{-1}\qquad\mathbf{-1}}& && \text{Subtract } 1 \text{ from both sides}\\
-13&=5x && \text{Focus on multiplication of } 5\\
\overline{\mathbf{5}}\quad\overline{\mathbf{5}}& && \text{Divide both sides by } 5\\
-\frac{13}{5}&=x && \text{Our Solution}
\end{aligned}$$

We can use this same process if there are parentheses in the problem. We will first distribute the coefficient in front of the parentheses, then clear the fractions. See here:

Example D) Solve: $\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$

Solution

	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Distribute.	$\frac{1}{2}\cdot y-\frac{1}{2}\cdot 5=\frac{1}{4}\cdot y-\frac{1}{4}\cdot 1$
Simplify.	$\frac{1}{2}y-\frac{5}{2}=\frac{1}{4}y-\frac{1}{4}$
Multiply by the LCD, four.	$4\left(\frac{1}{2}y-\frac{5}{2}\right)=4\left(\frac{1}{4}y-\frac{1}{4}\right)$
Distribute.	$4\cdot\frac{1}{2}y-4\cdot\frac{5}{2}=4\cdot\frac{1}{4}y-4\cdot\frac{1}{4}$
Simplify.	$2y-10=y-1$
Collect the variables to the left.	$2y-y-10=y-y-1$
Simplify.	$y-10=-1$
Collect the constants to the right.	$y-10+10=-1+10$
Simplify.	$y=9$

==

Worksheet: 1.4 Solving with Fractions

Solve each equation. Make sure any fractional solutions are in simplest form.

1) $\frac{3}{2}n - \frac{8}{3} = -\frac{29}{12}$

2) $-\frac{1}{2} = \frac{3}{2}k + \frac{3}{2}$

3) $\frac{3}{2}\left(\frac{7}{3}n + 1\right) = \frac{3}{2}$

4) $2b + \frac{9}{5} = -\frac{11}{5}$

5) $-\frac{5}{8} = \frac{5}{4}\left(r - \frac{3}{2}\right)$

6) $\frac{3}{5}(1 + p) = \frac{21}{20}$

7) $\frac{3}{2} - \frac{7}{4}v = -\frac{9}{8}$

8) $\frac{4}{5}(x + 15) = 3$

9) $\frac{x}{5} + \frac{3}{4} = \frac{x}{2} + \frac{3}{5}$

10) $\frac{2x-1}{4} = -3$

11) $\frac{3x-4}{5} = \frac{x+1}{2}$

==

Section 1.5 Formulas

Learning Objectives:

In this section you will:

- Solve linear formulas for a specific variable

Solve a Formula for a Specific Variable

We have all probably worked with some geometric formulas in our study of mathematics. Formulas are used in so many fields, it is important to recognize formulas and be able to manipulate them easily.

It is often helpful to solve a formula for a specific variable. If you need to put a formula in a spreadsheet, you must solve it for a specific variable first. We isolate that variable on one side of the equal sign with a coefficient of one and all other variables and constants are on the other side of the equal sign.

When solving formulas for a variable we need to focus on the one variable we are trying to solve for, all the others are treated just like numbers. This is shown in the following example. Two parallel problems are shown, the first is a normal one-step equation, the second is a formula that we are solving for x.

Example A) Solve for x:

$$\begin{array}{ccl} 3x = 12 & wx = z & \text{In both problems, } x \text{ is multiplied by something} \\ \overline{\mathbf{3}} \quad \overline{\mathbf{3}} & \overline{\boldsymbol{w}} \quad \overline{\boldsymbol{w}} & \text{To isolate the } x \text{ we divide by 3 or } w. \\ x = 4 & x = \dfrac{z}{w} & \text{Our Solution} \end{array}$$

We use the same process to solve $3x = 12$ for x as we use to solve $wx = z$ for x. Because we are solving for x we treat all the other variables the same way we would treat numbers. Thus, to get rid of the multiplication we divided by w. This same idea is seen in the following example.

Example B) Solve for n:

$$\begin{array}{rll} m + n = p & \text{for } n & \text{Solving for } n\text{, treat all other variables like numbers} \\ \underline{-\boldsymbol{m} \qquad -\boldsymbol{m}} & & \text{Subtract } m \text{ from both sides} \\ n = p - m & & \text{Our Solution} \end{array}$$

As p and m are not like terms, they cannot be combined. For this reason we leave the expression as $p - m$. This same one-step process can be used with grouping symbols.

Example C)
Solve for ‘y’: $8x + 7y = 15$

Solution

We will isolate y on one side of the equation.	$8x + 7y = 15$
Subtract $8x$ from both sides to isolate the term with y.	$8x - 8x + 7y = 15 - 8x$
Simplify.	$7y = 15 - 8x$
Divide both sides by 7 to make the coefficient of y one.	$\frac{7y}{7} = \frac{15 - 8x}{7}$
Simplify.	$y = \frac{15 - 8x}{7}$

Geometric formulas often need to be solved for another variable, too. In the next example, we are given the slope-intercept equation of a line and asked to solve for ‘*m*’, the slope.

Example D) Solve for ‘m’: y = mx + b

$y = mx + b$	for m	Solving for m, focus on addition first
$\underline{-\boldsymbol{b} \qquad -\boldsymbol{b}}$		Subtract b from both sides
$y - b = mx$		m is multipied by x.
$\overline{\boldsymbol{x}} \qquad \overline{\boldsymbol{x}}$		Divide both sides by x
$\frac{y-b}{x} = m$		Our Solution

It is important to note that we know we are done with the problem when the variable we are solving for is isolated or alone on one side of the equation and it does not appear anywhere on the other side of the equation.

Formulas often have fractions in them: first, identify the LCD and then multiply each term by the LCD. After we reduce there will be no more fractions in the problem so we can solve it like any general equation from there.

The formula $V = \frac{1}{3}\pi r^2 h$ is used to find the volume of a right circular cone when given the radius of the base and the height. In the next example, we will solve this formula for the height.

Example E)

Solve for h: $V = \frac{1}{3}\pi r^2 h$

Solution

Write the formula.			$V = \frac{1}{3}\pi r^2 h$
Remove the fraction on the right.			$3 \cdot V = 3 \cdot \frac{1}{3}\pi r^2 h$
Simplify.			$3V = \pi r^2 h$
Divide both sides by πr^2.			$\frac{3V}{\pi r^2} = h$

We could now use this formula to find the height of a right circular cone when we know the volume and the radius of the base, by using the formula $h = \frac{3V}{\pi r^2}$.

Example F)

$A = \pi r^2 + \pi r s$	for s	Solving for s, focus on what is added to the term with s
$\underline{-\pi r^2 \quad -\pi r^2}$		Subtract πr^2 from both sides
$A - \pi r^2 = \pi r s$		s is multipied by πr
$\overline{\pi r} \quad \overline{\pi r}$		Divide both sides by πr
$\frac{A - \pi r^2}{\pi r} = s$		Our Solution

Again, we cannot reduce the πr in the numerator and denominator because of the subtraction in the problem.

==

Worksheet: 1.5 Formulas

Solve each equation for the specified variable.

1) $a + c = b$ for c
2) $x - f = g$ for x
3) $S = L + 2B$ for L
4) $ax + b = c$ for x
5) $q = 6(L - p)$ for L
6) $S = \pi rh + \pi r^2$ for h
7) $R = aT + b$ for T
8) $rt = d$ for r
9) $V = lwh$ for w
10) $V = \frac{\pi r^2 h}{3}$ for h
11) $h = vt - 16t^2$ for v
12) $\frac{k-m}{r} = q$ for k

==

1.8 Applications: Number/Geometry

Learning Objectives

In this section, you will:

- Solve number problems.
- Solve basic geometry problems

I. Number Problems:

Word problems can be tricky. We will focus on some basic number problems, geometry problems, and parts problems. A few important phrases are described below that can give us clues for how to set up a problem.

- A number (or unknown, a value, etc.) often becomes our variable: ex. ‘x’
- Is (or other forms of is: was, will be, are, etc.) often represents equals ‘=’
  - ‘A number is 5’ becomes: x =5

Example A) Solve:

If 28 less than five times a certain number is 232. What is the number?

$$\begin{array}{rl} 5x-28 & \text{Subtraction is built backwards, multiply the unknown by 5} \\ 5x-28=232 & \text{Is translates to equals} \\ \underline{+\mathbf{28}+\mathbf{28}} & \text{Add 28 to both sides} \\ 5x=260 & \text{The variable is multiplied by 5} \\ \overline{\mathbf{5}}\quad\overline{\mathbf{5}} & \text{Divide both sides by 5} \\ x=52 & \text{The number is 52.} \end{array}$$

Example B) Solve:

Fifteen more than three times a number is the same as ten less than six times the number. What is the number

$$\begin{array}{rl} 3x+15 & \text{First, addition is built backwards} \\ 6x-10 & \text{Then, subtraction is also built backwards} \\ 3x+15=6x-10 & \text{Is between the parts tells us they must be equal} \\ \underline{-\mathbf{3x}\qquad-\mathbf{3x}} & \text{Subtract } 3x \text{ so variable is all on one side} \\ 15=3x-10 & \text{Now we have } a \text{ two} - \text{step equation} \\ \underline{+\mathbf{10}\qquad+\mathbf{10}} & \text{Add 10 to both sides} \\ 25=3x & \text{The variable is multiplied by 3} \\ \overline{\mathbf{3}}\quad\overline{\mathbf{3}} & \text{Divide both sides by 3} \\ \frac{25}{3}=x & \text{Our number is } \frac{25}{3} \end{array}$$

Example C) Solve: A sofa and a love seat together costs S444. The sofa costs double the love seat. How much do they each cost?

$$\begin{array}{rl} \text{Love Seat } x & \text{With no information about the love seat, this is our } x \\ \text{Sofa } 2x & \text{Sofa is double the love seat, so we multiply by 2} \\ S+L=444 & \text{Together they cost 444, so we add.} \\ (x)+(2x)=444 & \text{Replace } S \text{ and } L \text{ with labeled values} \\ 3x=444 & \text{Parenthesis are not needed, combine like terms } x+2x \\ \overline{\mathbf{3}}\quad\overline{\mathbf{3}} & \text{Divide both sides by 3} \\ x=148 & \text{Our solution for } x \\ \text{Love Seat } 148 & \text{Replace } x \text{ with 148 in the origional list} \\ \text{Sofa } 2(148)=296 & \text{The love seat costs \$148 and the sofa costs \$296.} \end{array}$$

==

Worksheet: 1.8 Number Problems

1. When five is added to three more than a certain number, the result is 19. What is the number?

2. If five is subtracted from three times a certain number, the result is 10. What is the number?

3. When 18 is subtracted from six times a certain number, the result is -42. What is the number?

4. A certain number added twice to itself equals 96. What is the number?

5. A number plus itself, plus twice itself, plus 4 times itself, is equal to -104. What is the number?

6. Sixty more than nine times a number is the same as two less than ten times the number. What is the number?

7. Eleven less than seven times a number is five more than six times the number. Find the number.

8. If Mr. Brown and his son together had S220, and Mr. Brown had 10 times as much as his son, how much money had each?

9. In a room containing 45 students there were twice as many girls as boys. How many of each were there?

10. An electrician cuts a 30 ft piece of wire into two pieces. One piece is 2 ft longer than the other. How long are the pieces?

11. The total cost for tuition plus room and board at State University is S2,584. Tuition costs S704 more than room and board. What is the tuition fee?

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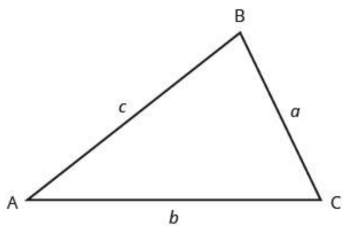
II. Geometry Problems

Another example of translating English sentences to mathematical sentences comes from geometry. We will discuss angles of triangles and perimeter problems.

Sum of the measures of the angles in a triangle:

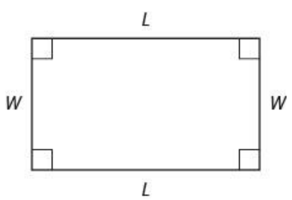
The plural of the word *vertex* is *vertices*. All triangles have three vertices: A, B, and C. The lengths of the sides are a, b, and c. The triangle is called by it1s vertices: △ABC.



The three angles of a triangle are related in a special way. The sum of their measures is 180°. Note that we read m∠A as "the measure of angle A." So in △ABC:

$$m\angle A+m\angle B+m\angle C=180°m\angle A+m\angle B+m\angle C=180°$$

Perimeter of rectangle: The opposite sides of a rectangle are the same length. We refer to one side of the rectangle as the length, *L*, and its adjacent side as the width, *W*.



The distance around this rectangle is L+W+L+W, or 2L+2W. This is the perimeter, *P*, of the rectangle.

$$P=2L+2W$$

Example D) **Solve:** The second angle of a triangle is double the first. The third angle is 40 less than the first. Find the three angles.

First x	With nothing given about the first we make that x
Second $2x$	The second is double the first,
Third $x-40$	The third is 40 less than the first
$F+S+T=180$	All three angles add to 180
$(x)+(2x)+(x-40)=180$	Replace F, S, and T with the labeled values.
$x+2x+x-40=180$	Here the parenthesis are not needed.
$4x-40=180$	Combine like terms, $x+2x+x$
$\underline{\mathbf{+40\ +40}}$	Add 40 to both sides
$4x=220$	The variable is multiplied by 4
$\overline{\mathbf{4}}\quad\overline{\mathbf{4}}$	Divide both sides by 4
$x=55$	Our solution for x
First 55	Replace x with 55 in the original list of angles
Second $2(55)=110$	Our angles are 55, 110, and 15
Third $(55)-40=15$	

Example E) **Solve:** The Perimeter of a rectangular outdoor patio is 54 ft. The length is 3ft greater than the width. What are the dimensions of the patio?

Solution The perimeter formula is standard: $P = 2L + 2W$. We have two unknown quantities, length and width. However, we can write the length in terms of the width as $L = W + 3$. Substitute the perimeter value and the expression for length into the formula. It is often helpful to make a sketch and label the sides as in **Figure 3**.

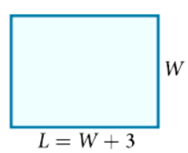


Figure 3

Now we can solve for the width and then calculate the length.

$$\begin{aligned} P &= 2L + 2W \\ 54 &= 2(W+3) + 2W \\ 54 &= 2W + 6 + 2W \\ 54 &= 4W + 6 \\ 48 &= 4W \\ 12 &= W \\ (12+3) &= L \\ 15 &= L \end{aligned}$$

The dimensions are $L = 15$ ft nd $W = 12$ ft.

Example F) Solve: The perimeter of a rectangle is 44. The length is 5 less than double the width. Find the dimensions.

Length x	We will make the length x
Width $2x-5$	Width is five less than two times the length
$P=2L+2W$	The formula for perimeter of a rectangle
$(44)=2(x)+2(2x-5)$	Replace P, L, and W with labeled values
$44=2x+4x-10$	Distribute through parenthesis
$44=6x-10$	Combine like terms $2x+4x$
$\underline{+\mathbf{10} \quad +\mathbf{10}}$	Add 10 to both sides
$54=6x$	The variable is multiplied by 6
$\overline{\mathbf{6}} \quad \overline{\mathbf{6}}$	Divide both sides by 6
$9=x$	Our solution for x
Length 9	Replace x with 9 in the origional list of sides
Width $2(9)-5=13$	The dimensions of the rectangle are 9 by 13.

<u>Worksheet: 1.8 Geometry Problems</u>

1) The second angle of a triangle is the same size as the first angle. The third angle is 12 degrees larger than the first angle. How large are the angles?
2) Two angles of a triangle are the same size. The third angle is 12 degrees smaller than the first angle. Find the measure the angles.
3) The third angle of a triangle is the same size as the first. The second angle is 4 times the third. Find the measure of the angles.
4) The second angle of a triangle is twice as large as the first. The measure of the third angle is 20 degrees greater than the first. How large are the angles?
5) The perimeter of a rectangle is 150 cm. The length is 15 cm greater than the width. Find the dimensions.
6) The Perimeter of a rectangle is 304 cm. The length is 40 cm longer than the width. Find the length and width.
7) The perimeter of a rectangle is 280 meters. The width is 26 meters less than the length. Find the length and width.
8) The perimeter of a college basketball court is 96 meters and the length is 14 meters more than the width. What are the dimensions?
9) The perimeter of a rectangle is 608 cm. The length is 80 cm longer than the width. Find the length and width.

==

1.9 Other Applications: Age, Sales Tax, Discount, and Commission Problems

Learning Objectives: In this section, you will:

- Set up a linear equation to solve an age problem
- Set up a linear equation to solve a Commission problem
- Set up a linear equation to solve Sales Tax problem
- Set up a linear equation to solve Discount problems.

I. Age Problems

An application of linear equations is what are called age problems. When we are solving age problems, we generally will be comparing the age of two people both now and in the future (or past). To help us organize and solve our problem we will fill out a table for each problem.

1. Fill in the now column. The person we know nothing about is x.
2. Fill in the future/past column by adding/subtracting the change to the now column.
3. Make an equation for the relationship in the future. This is independent of the table.
4. Replace variables in equation with information in future cells of table
5. Solve the equation for x, use the solution to answer the question

Example A) **Solve:** Carmen is 12 years older than David. Five years ago the sum of their ages was 28. How old are they now?

	Age Now	-5
Carmen		
David		

Five years ago is -5 in the change column.

	Age Now	-5
Carmen	$x+12$	
David	x	

Carmen is 12 years older than David. We don't know about David so he is x, Carmen then is $x+12$

	Age Now	-5
Carmen	$x+12$	$x+12-5$
David	x	$x-5$

Subtract 5 from now column to get the change

$C+D=28$	The sum of their ages will be 29. So we add C and D
$(x+7)+(x-5)=28$	Replace C and D with the change cells.
$x+7+x-5=28$	Remove parenthesis
$2x+2=28$	Combine like terms $x+x$ and $7-5$
$\underline{-\mathbf{2}\quad -\mathbf{2}}$	Subtract 2 from both sides
$2x=26$	Notice x is multiplied by 2
$\overline{\mathbf{2}}\quad\overline{\mathbf{2}}$	Divide both sides by 2
$x=13$	Our solution for x

	Age Now
Caremen	$13+12=25$
David	13

Replace x with 13 to answer the question
Carmen is 25 and David is 13

II. Mark-up/Discount problems

Mark-up/Sales tax formula Given the original cost of an item 'C', the mark-up/sales tax rate 'r', the selling price/total cost 'S' of the item including the mark-up/sales tax rate ('r': rate is a percentage and should be converted to a decimal) is given by:

$S = C + rC$,

Example B) **Solve sales tax problem:** Imagine that our food costs \$65 in a restaurant, and the sales tax is 8%. What is our total cost?

When paying for our meal at a restaurant, we do not pay just the price of the food. We also pay a percentage for sales tax. Then we would pay the original C=\$65 plus 8% of that \$65. The total cost would be:

$S = C + rC$ $\qquad$ $C = 65,\ r = 8\% = .08$

total cost = food cost + sales tax = 65 + 0.08(65) = \$70.20

Example C) Solve markup problem: A retailer acquired a laptop for \$2,015 and sold it for \$3,324.75. What was the percent markup?

Since the retailer acquired the laptop before it was sold, the \$2,015 price is the original. We can also consider that the retailer wants to make a profit, and this is a mark-up problem. We will use the mark-up formula,

$S = C + rC$, $\qquad$ where C = 2015 and S = 3324.75, to find the mark-up rate.

$3324.75 = 2015 + r \cdot 2015$ $\qquad$ subtract 2015 from both sides

$1309.75 = 2015r$ $\qquad$ divide both sides by 2015

$0.65 = r$

Since the mark-up rate is a percentage, then we convert r = 0.65 to a percentage. Hence, the mark-up rate is 65%

III. Commission problems

Commission is paid to an employee as an incentive to sell more. A commission is generally a percentage of sales.

Commission
To find your commission ‘C’, we multiply the sale price ‘S’ by the commission rate (‘r’: rate is a percentage and should be converted to a decimal) is given by:
$C = rS$

Example D) Solve: The Grey family's house was sold for \$200,000. How much the real estate agend will earn as commission? How much money will the family have after they pay their real estate agent the 5% commission?

$C = rS = 0.05(200{,}000) = \$10{,}000$ The real estate agent will get \$10,000

$\$200{,}000 - \$10{,}000 = \$190{,}000$

The family will get \$190,000 after they pay their real estate agent.

Discount formula
Given the regular cost of an item ‘R’, the discount rate ‘r’ (convert percent to decimal), the sale price ‘S’ of the item is given by
$S = R - rR$

Example E) Solve: Sue bought a sweater for \$307.70 after a 15% discount. How much was it before the discount?

Since we are looking for the price before the discount was taken and before Sue bought it on sale, our unknown is the regular price, R.
The price Sue actually paid for the sweater, \$307.70, is the sale price, S.
Also, since the sweater is on sale, we subtract from the regular price and we will use the discount formula, where R = 307.70 and r = 15% or 0.15.

$S = R - rR$

$307.70 = 1R - 0.15R$ combine like terms

$307.70 = 0.85R$ divide both sides by .85

$362 = R$

Thus, the regular price of the sweater is \$362.

Worksheet: 1.9 Age, Sales Tax, Discount, and Commission Problem.

1) A boy is 10 years older than his brother. In 4 years he will be twice as old as his brother. Find the present age of each.

2) A father is 4 times as old as his son. In 20 years the father will be twice as old as his son. Find the present age of each.

3) Find a) the sales tax and b) the total cost: Kim bought a winter coat for $250 in St. Louis, where the sales tax rate was 8.2% of the Purchase price.

4) Diego bought a new car for $ 26, 525. He was surprised that the dealer than added $2,387.25. what was the sales tax rate for this purchase?

5) What is the sale tax rate if a $ 7,594 purchase will have $569.55 of sales tax added to it?

6) Bob is a travel agent. He receives 7% commission when he books a cruise for a customer. How much commission will receive for booking a $3900 cruise?

7) Fernando receives 18% commission when he makes a computer sale. How much commission will he receive for selling a computer for $ 2,190?

8) Rikki earned $87 commission when she sold a $1450 stove. What rate of commission did she get?

9) Homer received $1140 commission when he sold a car for $28,500. What rate of commission did he get?

10) Marta bought a dishwasher that was on sale for $75 off. The original price of the dishwasher was &525. What was the sale price of the dishwasher?

11) Find a) the amount of discount and b) the sale price: Sergio bought a belt that was discounted 40% from an original price of $29.

12) Find 2) the amount of discount and b) the sale price: Oscar bought a barbecue grill that was discounted 65% from an original price of $ 395.

==

<u>**3.1 Solve and Graph Inequalities**</u>

In this section, you will:

- Solve for the solutions to linear inequalities.
- Graph and give interval notation for the solutions to linear inequalities

When we have an equation such as x = 4 we have a specific value for our variable. With inequalities we will give a range of values for our variable. To do this we will not use equals, but one of the following symbols:

$>$	Greater than
$\geqslant$	Greater than or equal to
$<$	Less than
$\leqslant$	Less than or equal to

It is often useful to **draw a picture** of the solutions to the inequality on a number line. We will start from the value in the problem and bold the lower part of the number line if the variable is smaller than the number, and bold the upper part of the number line if the variable is larger. The value itself we will mark with brackets, either) or (for less than or greater than respectively, and] or [for less than or equal to or greater than or equal to respectively.

Once the graph is drawn, we can quickly convert the graph into what is called **interval notation**. Interval notation gives two numbers, the first is the smallest value, the second is the largest value. If there is no largest value, we can use ∞ (infinity). If there is no smallest value, we can use − ∞ negative infinity. If we use either positive or negative infinity, we will always use a curved bracket for that value.

Example A)

Graph the inequality and give the interval notation

$x < 2$ Start at 2 and shade below

Use) for less than

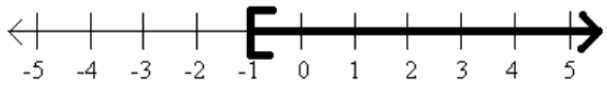
Our Graph

$(-\infty, 2)$ Interval Notation

Example B) Graph the inequality and give the interval notation

$y \geqslant -1$ Start at -1 and shade above

Use [for greater than or equal

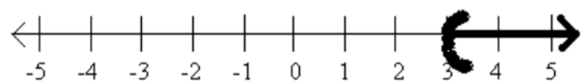


Our Graph

$[-1, \infty)$ Interval Notation

Example C)

Give the inequality for the graph:



Graph starts at 3 and goes up or greater. Curved bracket means just greater than

$x > 3$ Our Solution

Solving inequalities is very similar to solving equations with one exception:

If we multiply or divide by a negative number, the inequality symbol will need to reverse directions.

Example D) Solve and give the result in interval notation:

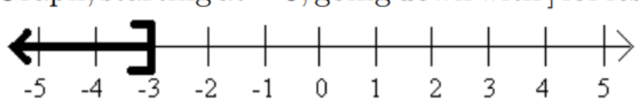
$5 - 2x \geqslant 11$ Subtract 5 from both sides

$\underline{-5 \qquad -5}$

$-2x \geqslant 6$ Divide both sides by -2

$\overline{-2} \quad \overline{-2}$ Divide by a negative $-$ flip symbol!

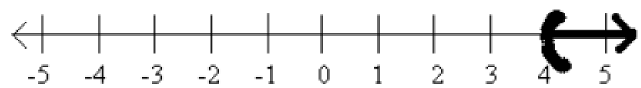
$x \leqslant -3$ Graph, starting at -3, going down with] for less than or equal to



$(-\infty, -3]$ Interval Notation

Example E) Solve and give the result in interval notation:

$$
\begin{array}{rl}
3(2x-4)+4x<4(3x-7)+8 & \text{Distribute} \\
6x-12+4x<12x-28+8 & \text{Combine like terms} \\
10x-12<12x-20 & \text{Move variable to one side} \\
\underline{-10x \qquad\quad -10x} & \text{Subtract } 10x \text{ from both sides} \\
-12<2x-20 & \text{Add 20 to both sides} \\
\underline{+20 \qquad\quad +20} & \\
8<2x & \text{Divide both sides by 2} \\
\overline{2} \quad \overline{2} & \\
4<x & \text{Be careful with graph, } x \text{ is larger!}
\end{array}
$$



$$(4,\infty) \quad \text{Interval Notation}$$

Note: The inequality symbol opens to the variable, this means the variable is greater than 4. So we must shade above the 4.

===

<u>Worksheet: 3.1 Solve and Graph Inequalities</u>

Draw a graph for each inequality and give interval notation.

1) $n > -5$

2) $n > 4$

3) $-2 \geqslant k$

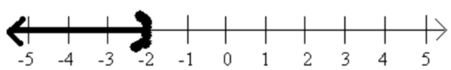
4) $1 \geqslant k$

5) $5 \geqslant x$

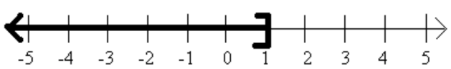
6) $-5 < x$

Write an inequality for each graph.

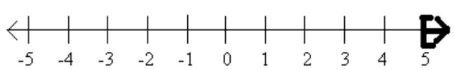
7)



8)



9)



Solve each inequality, graph each solution, and give interval notation.

10) $\frac{x}{11} \leq 10$

11) $2x + 5 \geq 10$

12) $-7x - 10 \leq 3x - 1$

13) $-2(p - 8) \leq 18$

14) $11 < 8 + 2x$

15) $8(n - 5) > -16$

16) $\frac{x}{4} > 5 + x$

17) $-\frac{1}{3}x \leq 6$

==

2.1: Graphing: Points and Lines

Learning Objectives

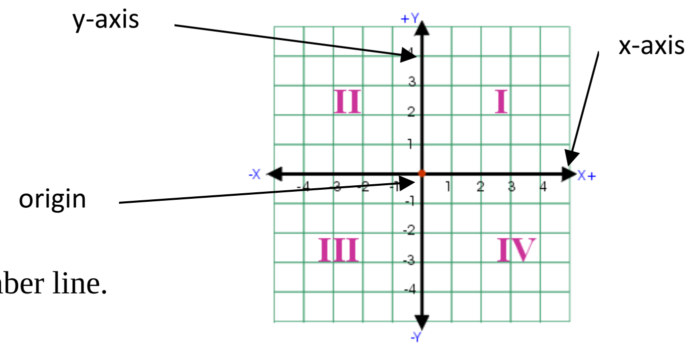
In this section, you will:

- Plot ordered pairs of numbers using xy coordinates
- Graph a linear equation by finding and plotting ordered pair solutions

I. Plot Ordered Pairs of Numbers Using XY Coordinates

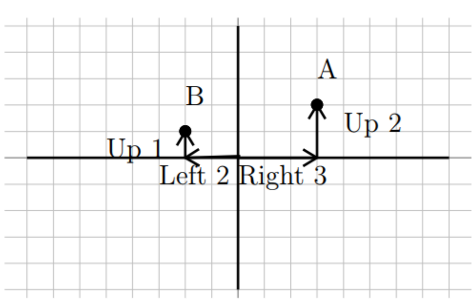
Vocabulary:

- **Coordinate plane:** The plane formed by two perpendicular lines called the x-axis and y-axis.
- **Quadrant:** The coordinate plane is divided into four regions. Each region is called a quadrant.



- **x-axis:** the horizontal number line.
- **y-axis:** the vertical number line.
- **Ordered pair:** a pair of numbers that represents a unique point in the coordinate plane. The first value is the x-coordinate and the second value is the y-coordinate.
  - Ex. 1: (2, 3) → 2 is the x-coordinate and 3 is the y-coordinate
- **Origin:** the center of the coordinate plane. It has coordinates (0, 0). It is the point where we always start when we are graphing.

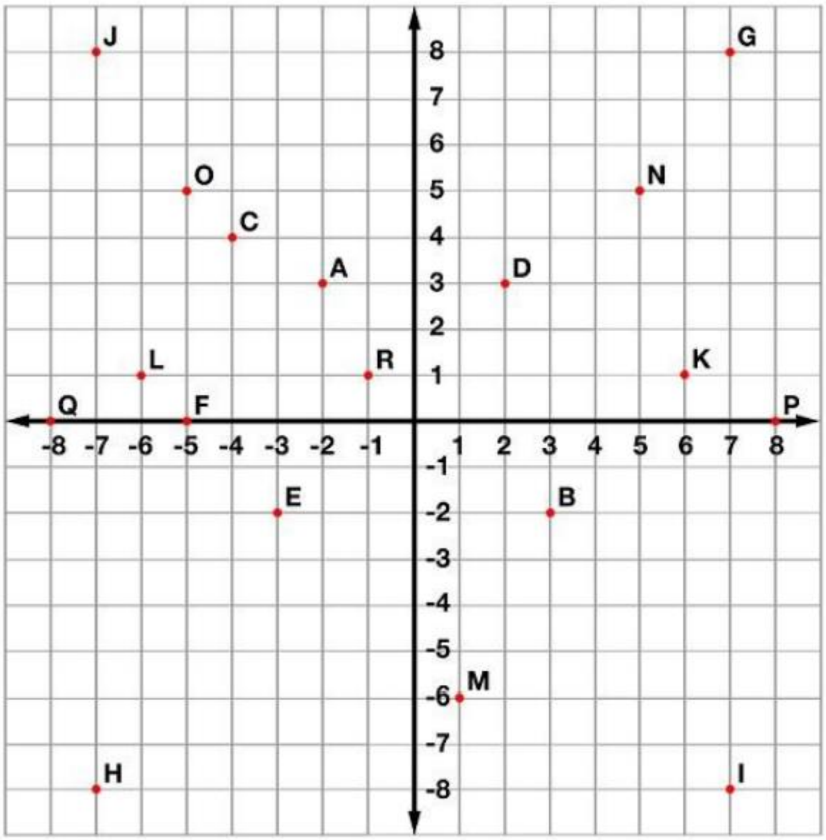
Example A) Graph the points A(3, 2), B (− 2, 1)



The first point, A is at (3, 2) this means $x = 3$ (right 3) and $y = 2$ (up 2). Following these instructions, starting from the origin, we get our point.

The second point, $B(-2, 1)$, is left 2 (negative moves backwards), up 1. This is also illustrated on the graph.

Worksheet: 2.1 Plot points in the Cartesian Coordinate System



Tell what point is located at each ordered pair.

1. **(3,-2)** _______ 2. **(2,3)** _______ 3. **(-5,5)** _______

4. **(-7,-8)** _______ 5. **(-4,4)** _______ 6. **(-5,0)** _______

Write the ordered pair for each given point.

7. **E** _______________ 8. **M** _______________ 9. **P** _______________

10. **G** _______________ 11. **Q** _______________ 12. **N** _______________

Plot the following points on the coordinate grid.

13. **S (-6,-3)** 14. **T (2,-4)** 15. **U (5,8)**

16. **V (4.5, -3.2)** 17. **W (0, -3)** 18. **X** $(\frac{7}{2}, \frac{5}{2})$

Identify the quadrant in which the point is located (ex 16-19).

19. **Point B in graph** _________ 20. **Point E in graph** ____________

21. **Point (-20, 50)** _________ 22. **Point (3.5, 100)** ____________

II. Graph a linear equation by finding and plotting ordered pair solutions

The main purpose of graphs is give a picture of the solutions to an equation. We will do this using a table of values.

To graph linear equations (using T-tables):

1. **Make a T-table that contains <u>at least 3 ordered pairs</u>**
   a. **Choose whatever numbers you want for "x", but keep it simple**
   b. **Substitute a value for "x", solve for "y" and fill in the table**
2. **Make the graph**
   a. **The straight line shows all possible solutions to the equation**
   b. **If points don't make straight line, double check the calculations in step #1 and the plotting of ordered pairs**

<u>Example B</u>) Graph: $y = 3x - 2$

T-Table

x	y
0	-2
1	1
2	4

Use $x = 1$

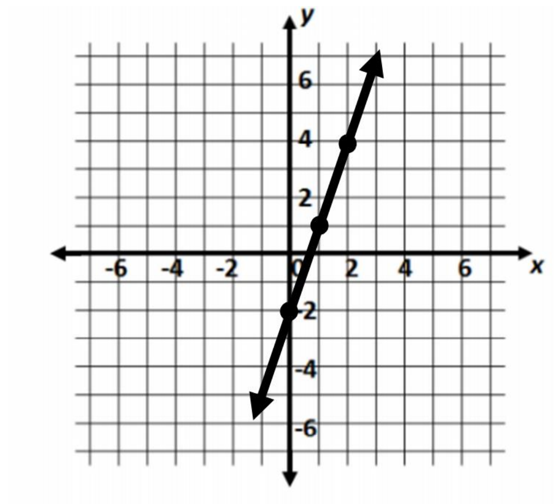
$y = 3(1) - 2$
$y = 3 - 2$
$y = 1$

Use $x = 0$

$y = 3(0) - 2$
$y = 0 - 2$
$y = -2$

Use $x = 2$

$y = 3(2) - 2$
$y = 6 - 2$
$y = 4$



<u>Example C</u>) Graph: $y = -2x + 5$

T-Table

x	y
0	5
1	3
2	1

$y = -2x + 5$

Use $x = 1$

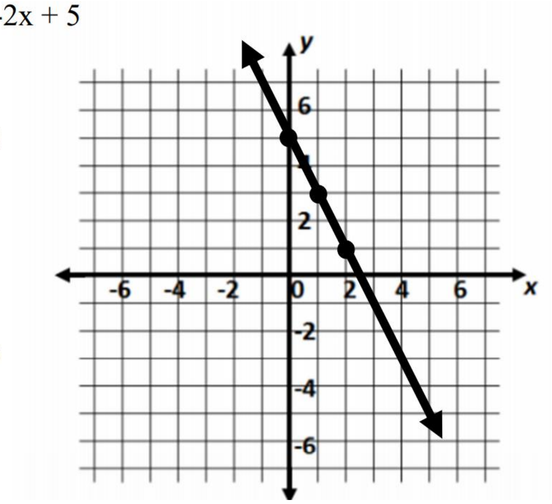
$y = -2(1) + 5$
$y = -2 + 5$
$y = 3$

Use $x = 0$

$y = -2(0) + 5$
$y = 0 + 5$
$y = 5$

Use $x = 2$

$y = -2(2) + 5$
$y = -4 + 5$
$y = 1$



Worksheet: 2.1 Graphing

Complete the table to find solutions to each linear equation.

1. y = −12x+3

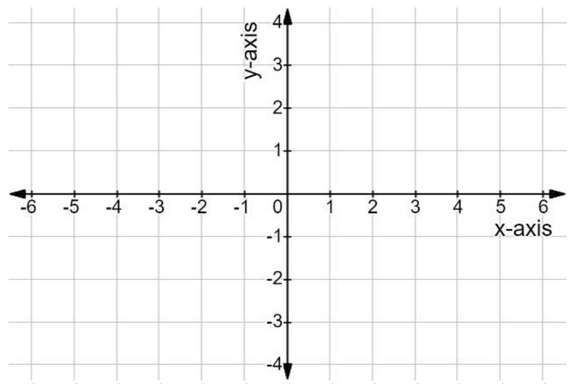
x	y	(x,y)
0		
4		
-2		

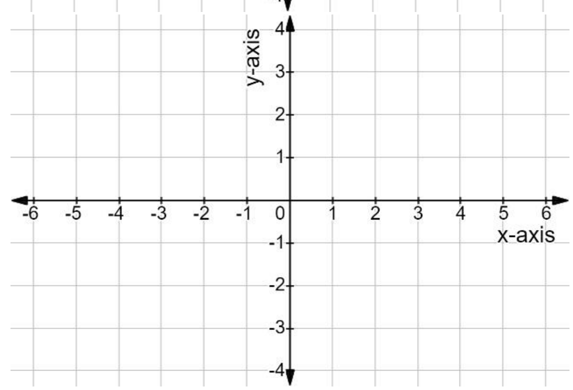
2. 3x + 2y = 6

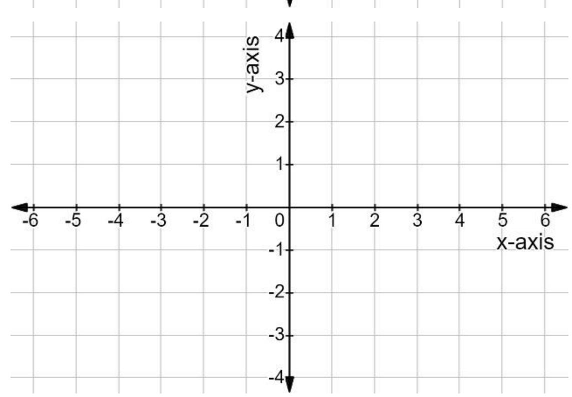
x	y	(x,y)
0		
	0	
-2		

Graph by plotting points.

3. y = -3x
4. y = 2x – 4
5. y = $\frac{1}{2}x + 3$
6. x – y = 6
7. 3x – 2y = 6
8. y = 5
9. x + 3 = 0







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2.2 Slope

Learning Objectives: In this section, you will:

- Find the slope of a line given a graph or two points
- Find x- and y-intercepts

Slope

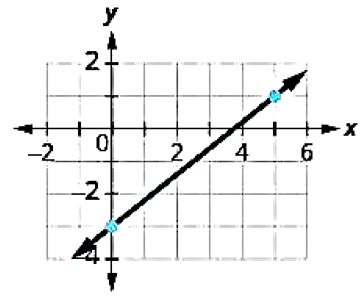
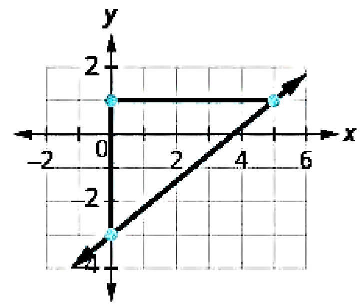
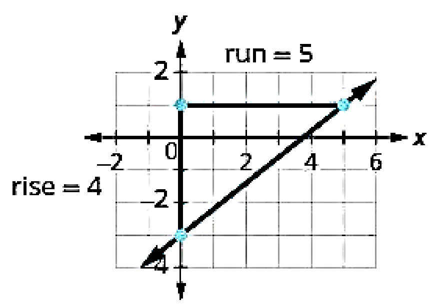
In everyday life a slope is in the pitch of a roof, the incline of a road, and the slant of a ladder leaning on a wall. In math, the slope defines steepness.
Slope = distance moved vertically divided by the distance moved horizontally
Easier to remember: **Slope = rise divided by run**.

We can find slope graphically and we can find the slope of the line given two points on the line

I. Finding slope graphically

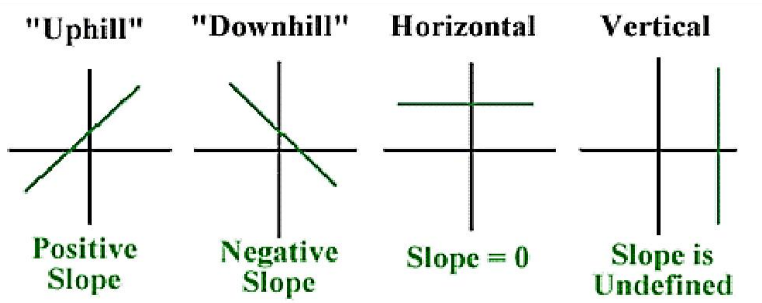
HOW TO: Find the slope of a line from its graph using $$\boldsymbol{slope = m = \frac{rise}{run}}$$
Step 1. Locate two points on the line whose coordinates are integers.
Step 2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
Step 3. Count the rise and the run on the legs of the triangle.
Step 4. Take the ratio of rise to run to find the slope: $m = \frac{rise}{run}$

Example A) Find the slope of the line shown.

Step 1. Locate two points on the graph whose coordinates are integers.	Mark (0, –3) and (5, 1).	
Step 2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.	Starting at (0, –3), sketch a right triangle to (5, 1).	
Step 3. Count the rise and the run on the legs of the triangle.	Count the rise. Count the run.	 The rise is 4. The run is 5.
Step 4. Take the ratio of rise to run to find the slope.	Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
$m = \frac{\text{rise}}{\text{run}}$	Substitute the values of the rise and run.	$m = \frac{4}{5}$ The slope of the line is $\frac{4}{5}$. This means that y increases 4 units as x increases 5 units.

Quick Guide to Slope

The direction of a line reveals if the slope is negative or positive. A horizontal line has a slope of 0 and a vertical line has an undefined slope.



II. **Finding the slope of the line given to points on the line**

Sometimes we'll need to find the slope of a line between two points when we don't have a graph. There is a way to find the slope without graphing.

We will use two points: (x_1, y_1) to identify the first point and (x_2, y_2) to identify the second point.

Slope of a Line

The slope m of the line containing the points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{rise}{run} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$x_2 \neq x_1$$

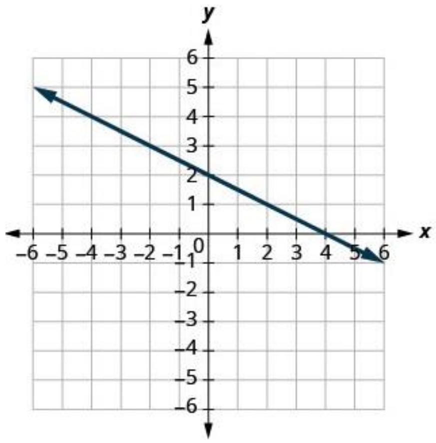
Example B) Find the slope of the line through (4, –3) and (2, 2).

Step 1: Label your points	Let (x_1, y_1) be (4, –3) and (x_2, y_2) be (2, 2)
Step 2: Use the slope formula and substitute the value	$m = \frac{rise}{run} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$ $x_2 \neq x_1$ $m = \frac{2-(-3)}{2-4} = \frac{5}{-2}$ Note: If we let (x_1, y_1) be (2, 2) and (x_2, y_2) be (4, –3), then we get the same result. $m = \frac{-3-2}{4-2} = \frac{-5}{2}$

III. Find x- and y-intercepts

X- and Y-intercepts
The points where a line crosses the *x*- axis and the *y*- axis are called the **intercepts of a line**.
The *x*- intercept is the point (a,0) where the line crosses the *x*- axis (y-coordinate is 0).
The *y*- intercept is the point (0,b) where the line crosses the *y*- axis (x-coordinate is 0).

Example C) Find the *x*- and *y*- intercepts on the graph.



X-int: line crosses x-axis: (4, 0)
Y-int: line crosses y-axis: (0, 2)

FIND THE *X*- AND *Y*- INTERCEPTS FROM THE EQUATION OF A LINE

To find:
- the *x*- intercept of the line, let y=0 and solve for x.
- the *y*- intercept of the line, let x=0 and solve for y.

Example D) Find the *x*- and *y*- intercepts from the equation: $2x-3y = 12$

- *x*- intercept of the line, let $y = 0$:
  $2x - 3(0) = 12$
  $2x = 12$ divide by 2
  $x = 6$ so x-int. is (6,0)

- *y*- intercept of the line, let $x = 0$:
  $2(0) – 3y = 12$
  $-3y = 12$ divide by -3
  $y = -4$ so y-int. is (0, -4)

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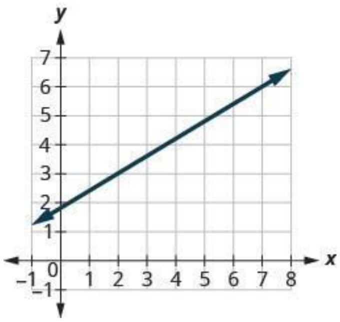
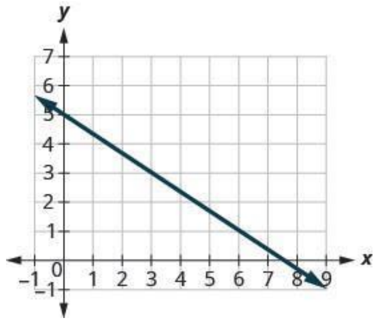
<u>Worksheet: 2.2 Slope</u>

Find the slope of the lines.

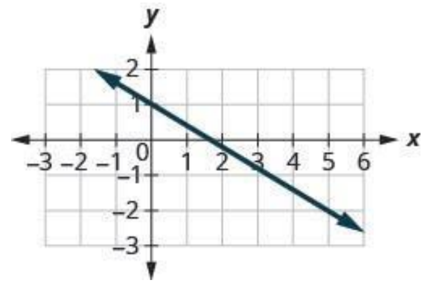
1.



2.



3.

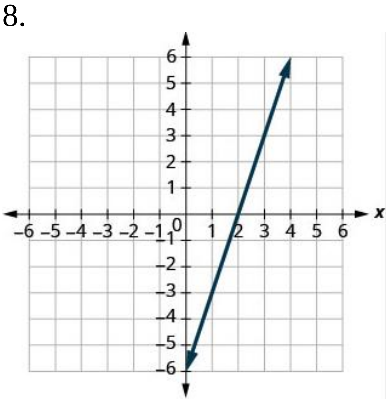


Find the slope of the line through each pair of points.

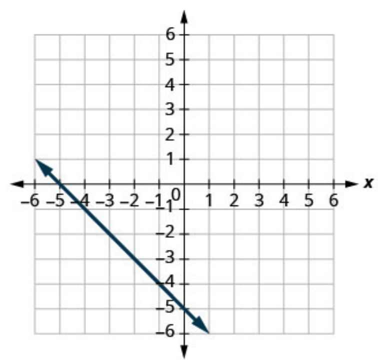
4. (13,15), (2,10)
5. (9,-6), (2,10)
6. (-16,2), (15, -10)
7. (-18, -5), (5,11)

Find the *x*- and *y*- intercepts on the graph.

8.



9.



Find the *x*- and *y*- intercepts from the equations:

10. x – 4y =20
11. 3x + 5y = -15
12. y = 3x -12
13. y = -x
14. x = 5

2.3 Graphing: Slope Intercept Form

Learning Objectives: In this section you will:

1) Give the equation of a line with a known slope and y-intercept.

A. Slope-Intercept Form

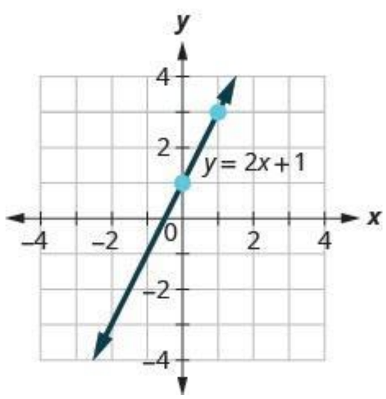
When graphing a line, one method is to make a table of values. If we can identify some properties of the line, we may be able to make a graph much quicker and easier.

One such method is finding the slope and the y-intercept of the equation. The slope can be represented by 'm' and the y intercept, where it crosses the axis and x = 0, can be represented by (0, b) where 'b' is the value where the graph crosses the vertical y-axis. Any other point on the line can be represented by (x, y). Using this information, we will look at the slope formula and solve the formula for y.

SLOPE-INTERCEPT FORM OF AN EQUATION OF A LINE
The **slope–intercept form** of an equation of a line with slope '**m**' and *y*-intercept, **(0,b)** is,

$$\mathbf{y = mx+b}$$

Example A) Use the graph to find the slope and *y*-intercept form of the line.



To find the slope of the line, we need to choose two points on the line. We'll use the points (0,1) and (1,3).

Find the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{1}$
	$m = 2$
Find the *y*-intercept of the line.	The *y*-intercept is the point (0, 1).
We found slope $m = 2$ and y-intercept (0, 1).	$y = 2x + 1$ $y = mx + b$

Example B) Identify the slope and *y*-intercept of the line with equation $y = -3x + 5$

Solution: We compare our equation to the slope–intercept form of the equation.

	$y = mx + b$
Write the equation of the line.	$y = -3x + 5$
Identify the slope.	$m = -3$
Identify the *y*-intercept.	*y*-intercept is $(0, 5)$

Example C) Identify the slope and *y*-intercept of the line with equation $x + 2y = 6$

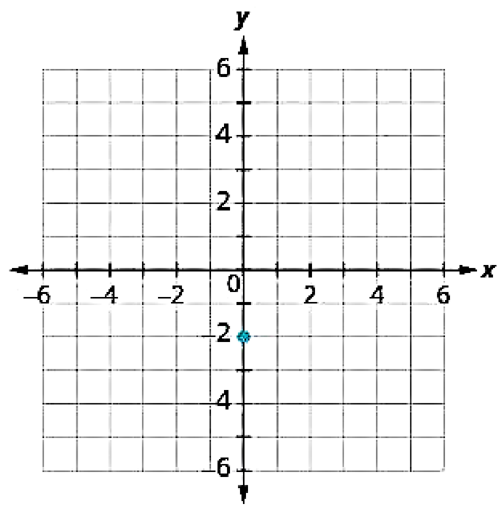
Solution

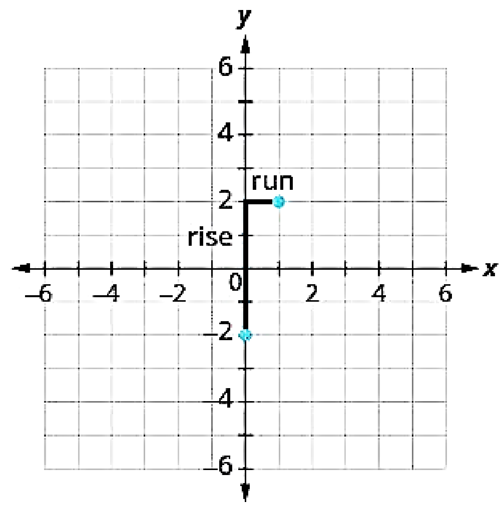
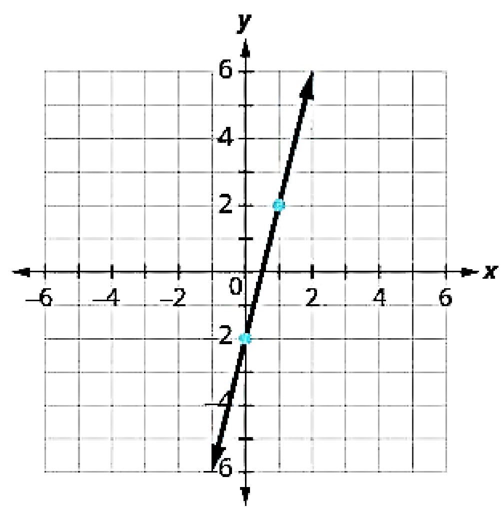
This equation is not in slope–intercept form. In order to compare it to the slope–intercept form we must first solve the equation for y.

Solve for *y*.	$x + 2y = 6$
Subtract *x* from each side.	$2y = -x + 6$
Divide both sides by 2.	$\frac{2y}{2} = \frac{-x+6}{2}$
Simplify.	$\frac{2y}{2} = \frac{-x}{2} + \frac{6}{2}$
Simplify.	$y = -\frac{1}{2}x + 3$
Write the slope–intercept form of the equation of the line.	$y = mx + b$
Write the equation of the line.	$y = -\frac{1}{2}x + 3$
Identify the slope.	$m = -\frac{1}{2}$
Identify the *y*-intercept.	*y*-intercept is $(0, 3)$

Example D) Graph the line of the equation y = 4x − 2 using its slope and *y*-intercept.

Solution:

Step 1. Find the slope–intercept form of the equation.	This equation is in slope–intercept form.	$y = 4x - 2$
Step 2. Identify the slope and *y*-intercept.	Use $y = mx + b$ Find the slope. Find the *y*-intercept.	$y = mx + b$ $y = 4x + (-2)$ $m = 4$ $b = -2$, $(0, -2)$
Step 3. Plot the *y*-intercept.	Plot (0, –2).	
Step 4. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = 4$ $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$ rise = 4 run = 1

Step 5. Starting at the *y*-intercept, count out the rise and run to mark the second point.	Start at (0, –2) and count the rise and the run. Up 4, right 1.	
Step 6. Connect the points with a line.	Connect the two points with a line.	

Note: If a line is not in slope-intercept form, solve for 'y' to find the slope-intercept form and you can graph it using the method above.

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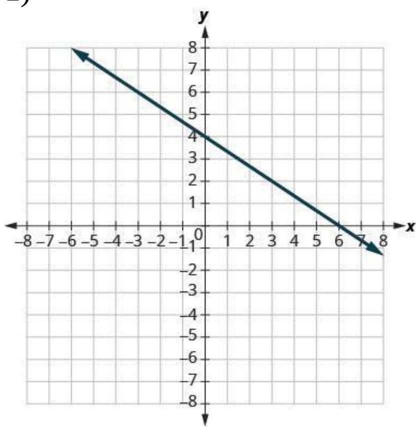
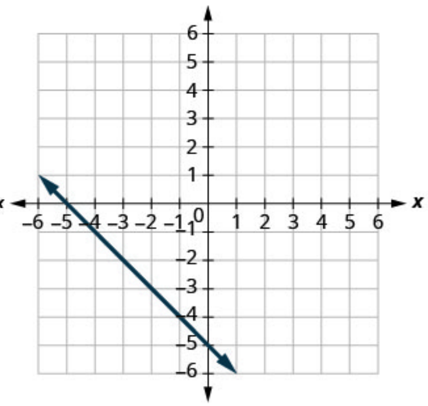
Worksheet: 2.3 Graphing: Slope Intercept Form

Use the graphs to find the slope and *y*-intercept of the lines.

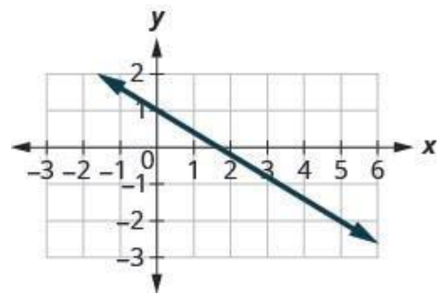
1)



2)



3)



Identify the slope and *y*-intercept of each line.

4) $y = 53x - 6$

5) $4x - 5y = 8$

6) $y = -4x + 9$

Write the slope-intercept form of the equation of each line given the slope and the y-intercept.

7) Slope = 2, y-intercept = 5

8) Slope = 1, y-intercept = − 4

9) Slope = − 3, y-intercept = − 1

10) Slope = 13 , y-intercept = 0

Graph the line of each equation using its slope and *y*-intercept.

11) $y = -x - 1$

12) $y = 3x + 2$

13) $4x - 3y = 12$

14) $x - 2y = 0$

2.4 Point-Slope Form

Learning Objectives:

In this section you will:

- Give the equation of a line with a knowns slope and point

We have two options for finding an equation of a line: slope–intercept or point–slope.

Write an equation of the line given a point and slope
The slope-intercept form is the simplest but requires us to know the y-intercept and slope. Sometimes we only know one or more points (that are not the y-intercept). In such a case we must use a different formula instead of slope intercept form. The formula of the equation we will use when the y-intercept is not given is called a **point-slope form** (as we will be using a random point and a slope).

Derivation of the formula

The slope of an equation is 'm', and a specific point on the line be (x1, y1), and any other point on the line be (x, y). We can use the slope formula to make a second equation.

Recall slope formula: m, (x1, y1), (x, y)

$$m = \frac{rise}{run}$$

$\frac{y2-y1}{x2-x1} = m$ Plug in the values

$\frac{y-y1}{x-x1} = m$ Multiply both side by$(x - x1)$

$y - y1 = m(x - x1)$ Our solution/formula

This is the point slope formula that requires one ordered pair and formula for the equation of a line. We can easily plug in values in this formula.

Point-Slope Formula

$$\boldsymbol{y - y1 = m(x - x1)}$$

When using this formula, we need a slope 'm' and a point on the line P(x1, y1). If the slope is not given, then you must find the slope to use the formula.

Example A) **Find an Equation of a Line Given the Slope and a Point:** Find an equation of a line with slope m=$\frac{2}{5}$ that contains the point (10,3). Write the equation in slope–intercept form.

Step 1. Identify the slope.	The slope is given.	$m = \frac{2}{5}$
Step 2. Identify the point.	The point is given.	$\begin{pmatrix} x_1 & y_1 \\ 10, & 3 \end{pmatrix}$
Step 3. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.		$y - y_1 = m(x - x_1)$ $y - 3 = \frac{2}{5}(x - 10)$
	Simplify.	$y - 3 = \frac{2}{5}x - 4$
Step 4. Write the equation in slope–intercept form.		$y = \frac{2}{5}x - 1$

Example B) Find an equation of a horizontal line that contains the point (−1,2). Write the equation in point-slope form.

Solution: Every horizontal line has slope 0. Since we have a point and slope we can substitute the slope and point into the point–slope form, **y−y1=m(x−x1)**.

Identify the slope. $m = 0$

Identify the point. $\begin{pmatrix} x_1 & y_1 \\ -1, & 2 \end{pmatrix}$

Substitute the values into y−y1=m(x−x1).

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 0(x - (-1))$$

Simplify.

$$y - 2 = 0(x + 1)$$
$$y - 2 = 0$$
$$y = 2$$

Write in slope–intercept form: It is in *y*-form but could be written y=0x+2.

Write in point-slope form: y-2 = 0

It is a horizontal line.

Example C) Find an equation of a line that contains the point (2,- 3) with an undefined slope. Write the equation in slope–intercept form.

Solution: If the slope is undefined then it is a vertical line. For this we can use the equation of vertical line and substitute. x = b

Identify the slope	m = undefined
Identify the point	(2, -3)
Substitute the values into x = b	**x = 2**

Find an Equation of the Line Given Two Points

Sometimes we are given just two points and no slope. In that case we need the slope to write out the equation of line. Once we find the slope (using the given points), we can use that and one of the given points to find the equation.

Since we will know two points, it will make more sense to use the point–slope form. Slope intercept form requires a slope and a point. Let's take a look at a problem.

Example D) **Find an Equation of a Line Given Two Points** Find an equation of a line that contains the points (5,4) and (3,6). Write the equation in slope–intercept form.

Step 1. Find the slope using the given points.	To use the point-slope form, we first find the slope.	$m=\frac{y_2-y_1}{x_2-x_1}$ $m=\frac{6-4}{3-5}$ $m=\frac{2}{-2}$ $m=-1$
Step 2. Choose one point.	Choose either point.	$\begin{pmatrix} x_1 & y_1 \\ 5, & 4 \end{pmatrix}$
Step 3. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.	Simplify.	$y-y_1=m(x-x_1)$ $y-4=-1(x-5)$ $y-4=-1x+5$
Step 4. Write the equation in slope–intercept form.		$y=-1x+9$

In summary we can look at the table below to help us remember what formula to use when writing an equation of line.

To Write an Equation of a Line		
If given:	Use:	Form:
Slope ‘m’ and *y*-intercept (0, b)	slope–intercept	**y = mx+b**
Slope ‘m’ and a point: P (x1, y1)	point–slope	**y−y1=m(x−x1)**
Two points: P1 (x1, y1); P2 (x2, y2) Note: find slope ‘m’ first	point–slope	**y−y1=m(x−x1)**

<u>Worksheet: 2.3 Point-Slope Form</u>

1) Write the equation of the line through the point (3, -4) with a slope of $\frac{3}{5}$
2) Write the equation of the line through the point (-1, 4) with a slope of $-\frac{5}{4}$
3) Write the equation of the line through the point (2, 2) with a slope of -2
4) Write the equation of the line through the point (-6, 3) with a slope of 0.
5) Write the equation of a vertical line passing through the point (-6, 3).
6) Find the equation of a line that contains the points (5,4) and (3,6). This time around use the point (3,6) to write the equation.
7) Find an equation of a line containing the points (3,1) and (5,6).
8) Find an equation of a line containing the points (1,4) and (6,2).
9) Find an equation of a line containing the points (-5,4) and (-5,2).
10) Find an equation of a line containing the points (-2,3) and (5,3).

==

2.5 Parallel and Perpendicular Lines

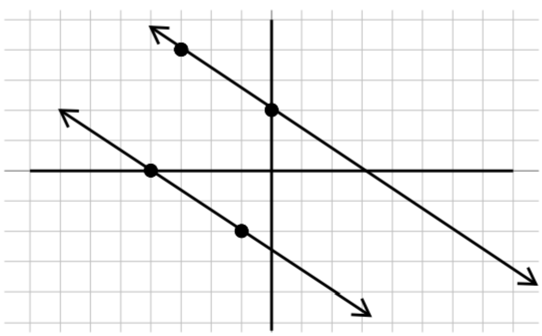
Learning Objectives: In this section you will:

- Determine whether the lines are parallel or perpendicular
- Write an equation of a line given a parallel or perpendicular line

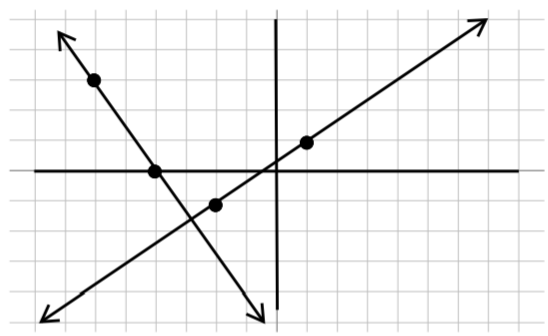
Parallel lines have the same slope. m1 = m2

Perpendicular lines have opposite (one +, one -) reciprocal (flipped fractions) slopes. $\boldsymbol{m2 = -\frac{1}{m1}}$

Example A) Find the slopes and decide whether the lines are parallel or perpendicular.



The above graph has two parallel lines. The slope of the top line is down 2, run 3, or $-\frac{2}{3}$. The slope of the bottom line is down 2, run 3 as well, or $-\frac{2}{3}$.



The above graph has two perpendicular lines. The slope of the flatter line is up 2, run 3 or $\frac{2}{3}$. The slope of the steeper line is down 3, run 2 or $-\frac{3}{2}$.

Example B)

Find the slope of a line perpendicular to $3x-4y=2$

$3x-4y=2$	To find slope we will put equation in slope $-$ intercept form
$\underline{-3x \qquad -3x}$	Subtract $3x$ from both sides
$-4y=-3x+2$	Put x term first
$\overline{-4} \quad \overline{-4} \; \overline{-4}$	Divide each term by -4
$y=\frac{3}{4}x-\frac{1}{2}$	The slope is the coefficient of x
$m=\frac{3}{4}$	Slope of first lines. Perpendicular lines have opposite reciprocal slopes
$m=-\frac{4}{3}$	Our Solution

Example C) Determine if the given set of Lines are parallel, perpendicular or neither.

$$y = -2x + 6 \;;\quad 2x + y = \; -4$$

To find out if the lines are parallel or perpendicular, we need to look at their slope.

Step 1: Identify the slope of line 1.	Slope-intercept form: the slope is given (coefficient of x). $y = -2x + 6$	$m = -2$
Step 2: Identify the slope of line 2.	The slope of this equation will have to be found. $2x + y = \; -4$	$2x + y = \; -4$
	To find the slope, arrange the equation to slope-intercept form by solving for y	$y = -2x \; - 4$
	In this form the slope is given (coefficient of x)	$m = -2$
Step 3: Compare the slopes of two lines.	Since both of the lines have same slopes, we can identify them as parallel lines.	**Parallel lines**

HOW TO: Find an equation of a line parallel or perpendicular to a given line.

1. Find the slope of the given line: ‘m’
2. Find the slope of the parallel or perpendicular line.
   Remember: parallel has same slope ‘m’, while perpendicular has opposite slope $-\frac{1}{m}$
3. Identify the point.
4. Substitute the values into the point–slope form, y−y1=m(x−x1).
5. Write the equation in slope–intercept form.

Example D) Find an equation of a line parallel to y=2x−3 that contains the point (−2,1). Write the equation in slope–intercept form.

Solution:

Step 1. Find the slope of the given line.	The line is in slope-intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the parallel line.	Parallel lines have the same slope.	$m_{\parallel} = 2$
Step 3. Identify the point.	The given point is, (–2, 1).	$\begin{pmatrix} x_1, & y_1 \\ -2, & 1 \end{pmatrix}$

Step 4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 1 = 2(x - (-2))$ $y - 1 = 2(x + 2)$ $y - 1 = 2x + 4$
Step 5. Write the equation in slope–intercept form.		$y = 2x + 5$

Example E) Find an equation of a line parallel to y=2x−3 that contains the point (−2,1). Write the equation in slope–intercept form.
Solution:

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the perpendicular line.	The slopes of perpendicular lines are negative reciprocals.	$m_\perp = -\frac{1}{2}$
Step 3. Identify the point.	The given point is, (–2, 1)	$\begin{pmatrix} x_1 & y_1 \\ -2, & 1 \end{pmatrix}$
Step 4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 1 = -\frac{1}{2}(x - (-2))$ $y - 1 = -\frac{1}{2}(x + 2)$ $y - 1 = -\frac{1}{2}x - 1$
Step 5. Write the equation in slope–intercept form.		$y = -\frac{1}{2}x$

Notes: Because a horizontal line is perpendicular to a vertical line we can say that no slope and zero slope are actually perpendicular slopes.

Worksheet: 2.5 Parallel and Perpendicular Lines

Find the slope of a line parallel to each given line.

1. $y = 2x + 4$
2. $y = -\frac{2}{3}x + 5$
3. $y = 4x - 5$
4. $y = -\frac{10}{3}x - 5$
5. $6x - 5y = 20$
6. $3x + 4y = -8$

Find the slope of a line perpendicular to each given line.

7. $x = 3$
8. $y = -\frac{1}{3}x$
9. $x - 3y = -6$
10. $y = -\frac{1}{2}x - 1$
11. $8x - 3y = -9$
12. $3x - y = -3$

13) Determine if the given lines are parallel, perpendicular or neither.

$$3x = 2y + 3$$
$$2x + 3y = 2$$

14) Determine if the given lines are parallel, perpendicular or neither.

$$x + 3y = 4$$
$$8x + 2y = 2$$

15) Determine if the given lines are parallel, perpendicular or neither.

$$9x = 16 - 3y$$
$$16 - 4y = 12x$$

Find the equation of the line given the following. Write the answer in slope-intercept form.

16) through: $(4, -3)$, parallel to $y = -2x$

17) through: $(-5, 2)$, parallel to $y = \frac{3}{5}x$

18) through: $(-3, 1)$, parallel to $y = -\frac{4}{3}x - 1$

19) through: $(4, -2)$, parallel to $y = -11$

20) through: $(4, 3)$, perpendicular to $x + y = -1$

21) through: $(-3, -5)$, perpendicular to $x + 2y = -4$

22) through: $(5, 2)$, perpendicular to $x = 0$

23) through: $(4, -3)$, perpendicular to $y = \frac{1}{2}x - 6$

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4.1 Solving Systems of Equations by Graphing

Learning Objectives:

In this section you will:

- To solve systems of equation by graphing and identifying the point of intersection

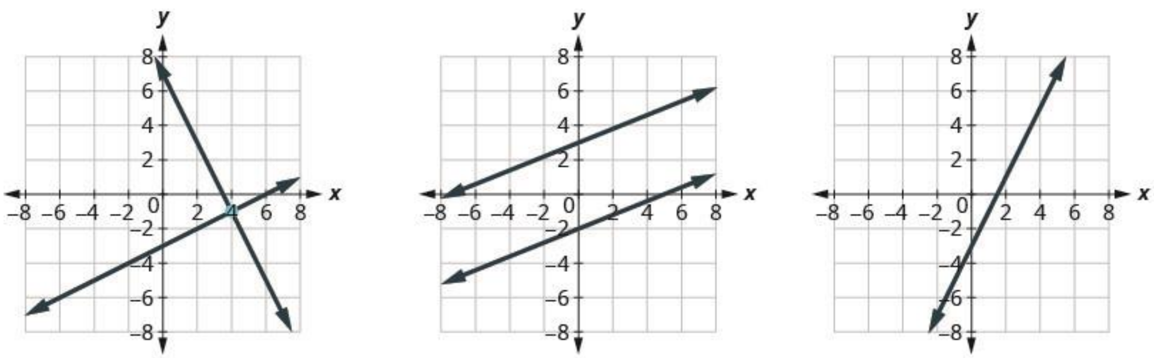
So far, we have solved linear equations in one variable like $8x = -2x + 20$.

When we have several equations, we call these a **system of linear equations.** To solve for two variables such as x and y we will need two equations. We are looking for a solution i.e the ordered pair that works in both equations.

Remember the graph of a linear equation is a line. For a system of two equations, we will graph two lines. By finding what the lines have in common, we'll find the solution to the system.

Most linear equations in one variable have one solution, but we saw that some equations, called contradictions, have no solutions and for other equations, called identities, all numbers are solutions.

There are three possible cases, as shown below



The lines intersect.
Intersecting lines have one point in common.
There is one solution to this system.

The lines are parallel.
Parallel lines have no points in common.
There is no solution to this system.

Both equations give the same line.
Because we have just one line, there are infinitely many solutions.

First, we decide whether a given ordered pair is the solution to the systems of linear equation.

Example A) Is (2,1) the solution to the system**?**

$$3x - y = 5$$
$$x + y = 3$$

$(2,1)$	Identify x and y from the orderd pair
$x = 2,\ y = 1$	Plug these values into each equation
$3(2) - (1) = 5$	First equation
$6 - 1 = 5$	Evaluate
$5 = 5$	True
$(2) + (1) = 3$	Second equation, evaluate
$3 = 3$	True

As we found a true statement for both equations using (2,1) we know that (2,1) is the solution. The goal of is to find that ordered pair for each given problem.

Example B) Is (-5, -3) the solution to the system?

$$2x + y = -7$$
$$3x + 2y = -9$$

(-5, -3)	Identify x and y from the ordered pair
X = -5, y = -3	Plug these values into each equation
2(-5)+(-3) = -7	First equation
-10-3 = -7	Evaluate
-13= -7	False, (-5, -3) is not a solution

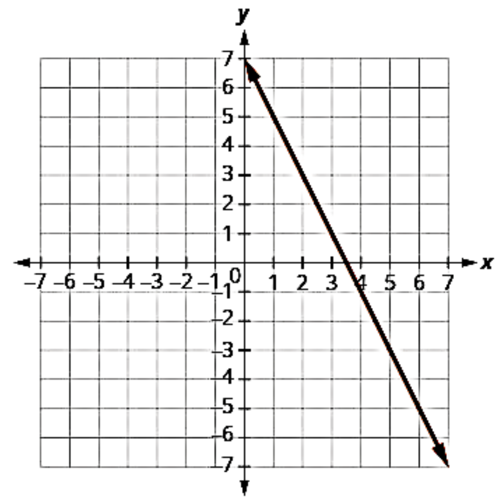
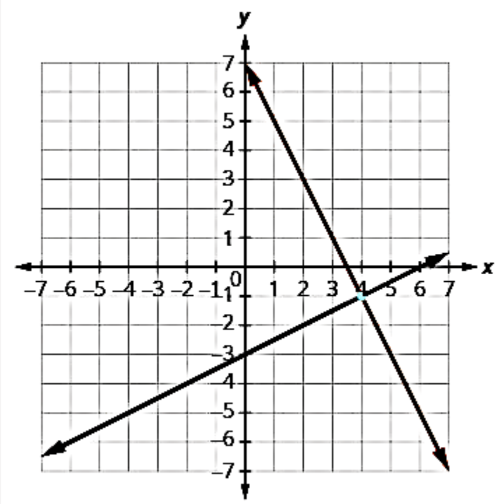
Since in this case the ordered pair is not a solution to the first equation there is no need to check it for the second one. As for the ordered pair to be the solution for the entire system it must work for both of the given equation.

Solving systems with a graph

We should graph two lines on the same coordinate plane to see the solutions of both equations. Our solution is a solution for both lines, this would be where the lines intersect.

Example C) Solve the System of Linear Equations by Graphing

$$2x + y = 7$$
$$x - 2y = 6$$

Step 1. Graph the first equation.	To graph the first line, write the equation in slope–intercept form. $2x + y = 7$ $y = -2x + 7$ $m = -2$ $b = 7$	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$ 
Step 2. Graph the second equation on the same rectangular coordinate system.	To graph the second line, use intercepts. $x - 2y = 6$ $(0, -3)$ $(6, 0)$	
Step 3. Determine whether the lines intersect, are parallel, or are the same line.	Look at the graph of the lines.	The lines intersect.

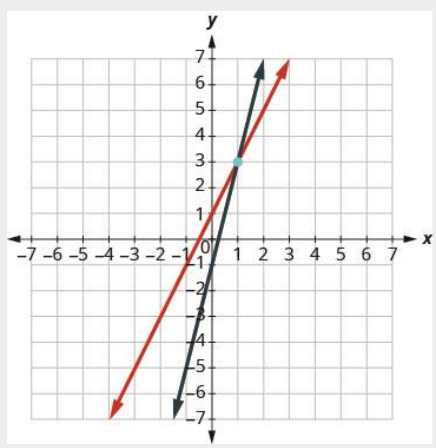
Step 4. Identify the solution to the system. If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system. If the lines are parallel, the system has no solution If the lines are the same, the system has an infinite number of solutions.	Since the lines intersect, find the point of intersection. Check the point in both equations.	The lines intersect at (4, –1). $2x + y = 7$ $2(4) + (-1) \stackrel{?}{=} 7$ $8 - 1 \stackrel{?}{=} 7$ $7 = 7 ✓$ $x - 2y = 6$ $4 - 2(-1) \stackrel{?}{=} 6$ $6 = 6 ✓$ The solution is (4, –1).

Example D) Solve the System of Linear Equations by Graphing

$$y = 2x + 1$$
$$y = 4x - 1$$

Both equations in this system are in slope-intercept form, so we will use their slopes and y-intercepts to graph them.

Find the slope and *y*-intercept of the first equation.	$y = 2x + 1$ $m = 2$ $b = 1$
Find the slope and *y*-intercept of the first equation.	$y = 4x - 1$ $m = 4$ $b = -1$
Graph the two lines.	
Determine the point of intersection.	The lines intersect at (1, 3).



Since both lines intersect at the ordered pair (1,3), it is the solution for the system.

So far, the lines intersected, and the solution was one point. In the next two examples, we'll look at a system of equations that has no solution and at a system of equations that has an infinite number of solutions.

Example E) Solve the System of Linear Equations by Graphing

$$y = \frac{3}{2}x - 4$$
$$y = \frac{3}{2}x + 1$$

Solution:

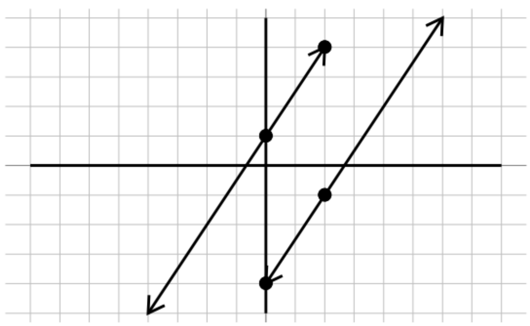
$$y = \frac{3}{2}x - 4$$
$$y = \frac{3}{2}x + 1$$

Identify slope and $y -$ intercept of each equation

First: $m = \frac{3}{2}, b = -4$

Second: $m = \frac{3}{2}, b = 1$

Now we can graph both equations on the same plane



To graph each equation, we start at the y-intercept and use the slope $\frac{\text{rise}}{\text{run}}$ to get the next point and connect the dots.

The two lines do not intersect! They are parallel! If the lines do not intersect we know that there is no point that works in both equations, there is no solution

$\varnothing$ No Solution

Note: We also could have noticed that both lines have the same slope. Remembering that parallel lines have the same slope we would have known there was no solution even without having to graph lines.

Example F) Solve the System of Linear Equations by Graphing

$$2x - 6y = 12$$
$$3x - 9y = 18$$

Solution:

$$\begin{array}{ll} 2x - 6y = 12 & 3x - 9y = 18 \\ \underline{-2x \qquad -2x} & \underline{-3x \qquad -3x} \\ -6y = -2x + 12 & -9y = -3x + 18 \\ \overline{-6} \quad \overline{-6} \quad \overline{-6} & \overline{-9} \quad \overline{-9} \quad \overline{-9} \\ y = \frac{1}{3}x - 2 & y = \frac{1}{3}x - 2 \end{array}$$

Subtract x terms

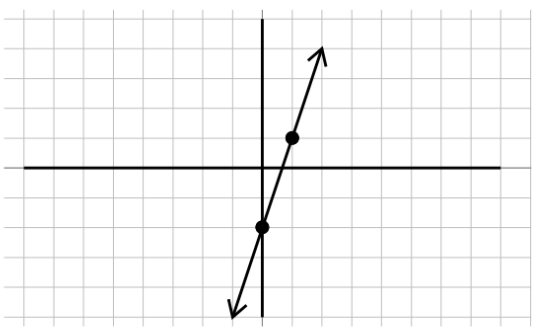
Put x terms first

Divide by coefficient of y

Identify the slopes and $y-$intercepts

$$\text{First: } m = \frac{1}{3}, b = -2$$
$$\text{Second: } m = \frac{1}{3}, b = -2$$

Now we can graph both equations together



To graph each equation, we start at the y-intercept and use the slope $\frac{\text{rise}}{\text{run}}$ to get the next point and connect the dots.

Both equations are the same line! As one line is directly on top of the other line, we can say that the lines "intersect" at all the points! Here we say we have infinite solutions

Once we have both equations in the slope-intercept form we can see that they both are the same equations. Moreover, by graphing we can see they both lie on top of each other giving us infinite number of solutions.

Determine the Number of Solutions of a Linear System:

Graph	Number of solutions
2 intersecting lines	1
Parallel lines	None
Same line	Infinitely many

==

Worksheet: 4.1 Solving Systems of Equations by Graphing and Identifying the Point of Intersection

1) Is (2, -4) is the solution to the system?

$$4x = 4 - y$$
$$2x = -12 - 4y$$

2) Is (-3, 1) is the solution to the system?

$$3x = 10 - y$$
$$4x = 15 - 3y$$

3) Is (-2, 0) is the solution to the system?

$$-5x + y = -2$$
$$-3x + 6y = -12$$

Solve the following systems of linear equations using graphing method:

4) $y = -2x + 1$
   $y = \frac{1}{2}x - 2$

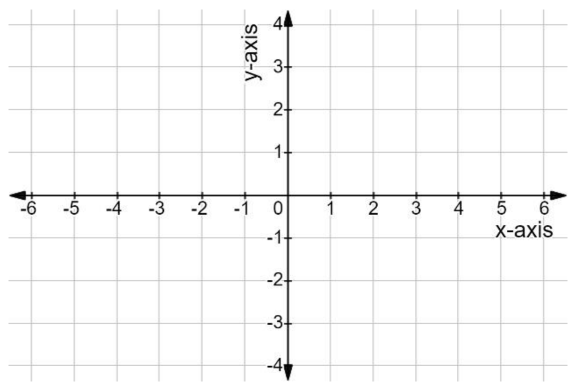
5) $y = -2x + 2$
   $x + 2y = -2$

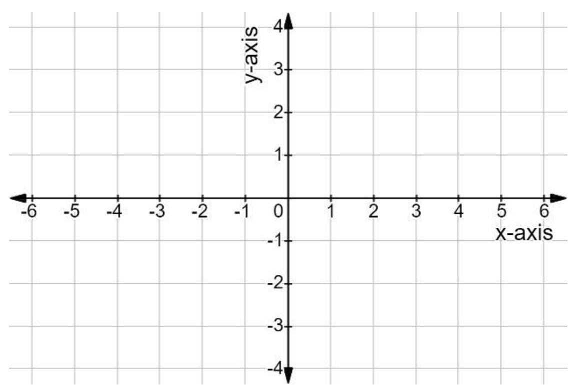
6) $2x + y = 1$
   $2x - y = 3$

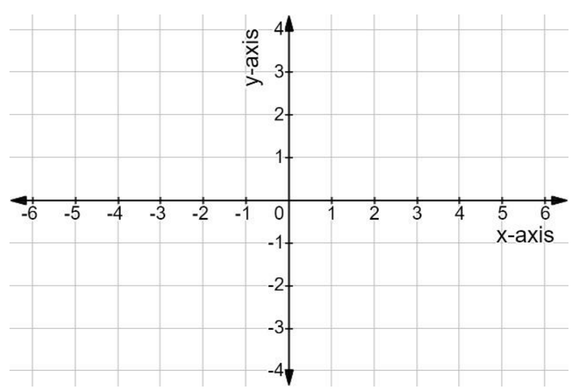
7) $3x - y = 1$
   $y = -4$

8) $2y + x = 4$
   $2y = -x + 4$

9) $5x - y = 4$
   $10x - 2y = 10$







4.2 Solving Systems by Substitution

Learning Objectives: In this section, you will:

- Solve systems of equations using substitution

A system of equations has multiple variables ('x', 'y' or others). If we can get it down to one variable, we can solve the system just like we have solved equations previously. Our goal is to take this:

$$Ax + By = C$$
$$Dx + Ey = F$$

and turn it into either a single equation with only 'x' or only 'y' in it.

Note: A, B, C, D, E, and F are just numbers.

Steps for substitution method
1. Solve one of the equations for either variable
2. Substitute the expression from Step 1 into the other equation
3. Solve the resulting equation
4. Substitute the solution in Step 3 into either of the original equations to find the other variable
5. Write the solution as an ordered pair
6. Check that the ordered pair is a solution to both original equations

Example A) Solve the system by substitution:

$$3x + y = -3 \qquad (1)$$
$$2x + 3y = 5 \qquad (2)$$

Solution:

Since the first equation has *y* with a coefficient of 1, we will solve the first equation for *y*

$$y = -3x - 3 \qquad (3)$$
$$2x + 3y = 5 \qquad (2)$$

We will then substitute what we have for *y* equation (3) into equation (2) and solve

$2x + 3(-3x - 3) = 5$	Substitution
$2x - 9x - 9 = 5$	Distribute
$-7x - 9 = 5$	Collect like terms
$-7x = 14$	Isolate the variable
$x = -2$	Divide away the coefficient

Now that we know what *x* is, we can substitute it into equation (3) to get *y*

$$y = -3(-2) - 3 = 6 - 3 = 3$$

So the solution is (2, 3). Remember we can always check our solution by plugging it into the original system.

Example B) Solve the system by substitution:

$$x + y = -4 \quad (1)$$
$$x = 2y + 5 \quad (2)$$

Solution:

Since the second equation (2) is already solved for *x*, we can go straight to plugging the solved form into equation (1)

$2y + 5 + y = -4$	Substitution
$3y + 5 = -4$	Collect like terms
$3y = -9$	Isolate the variable
$y = -3$	Divide away the coefficient

Now that we know what *y* is, we can substitute it into equation (2) to get *x*

$$x = 2(-3) + 5 = -6 + 5 = -1$$

So the solution is (1,3). Remember we can always check our solution by plugging it into the original system.

Special Cases: It is important to remember that we have three possible solutions for a system of equations:

1. The system has **one solution**
2. The system has **no solution**
   - variable from will cancel out, leaving just numerical values
   - **the numbers will not equal one another**, making a false statement
3. The system has **infinitely many solution**
   - variable from will cancel out, leaving just numerical values
   - **the numbers will be equal one another**, making a true statement

Example C) Solve the system by substitution:

$$x + 3y = 4 \quad (1)$$

$$-2x - 6y = 3 \quad (2)$$

Solution:

Since the first equation has *x* with a coefficient of 1, we will solve the first equation for *x*

$$x = 4 - 3y \quad (3)$$

We can now substitute equation (3) into equation (2)

$-2(4 - 3y) - 6y = 3$	Substitution
$-8 + 6y - 6y = 3$	Distribute
$-8 = 3$	Simplify

We see the variable *y* has cancelled, and the statement left is false since the two numbers are not equal. This means the system has **no solution**.

Worksheet: 4.2 Solving Systems by Substitution

Find the solutions to the systems:

1) $y = x + 2$
   $2x + y = -4$

2) $y = x + 2$
   $y = -2x + 2$

3) $x + 4y = 6$
   $-8x + y = -81$

4) $-x + y = -1$
   $4x - 3y = 6$

5) $x = -3y + 4$
   $2x + 6y = 8$

6) $2x + y = 5$
   $-8x - 4y = -24$

4.3 Solving Systems by Addition

Learning Objectives: In this section, you will:

- Solve systems of equations using the addition/elimination method

A system of equations has multiple variables. If we can get it down to one variable, we can solve the system just like we have solved equations previously. Our goal is to take this:

$$Ax + By = C$$
$$Dx + Ey = F$$

and turn it into either a single equation with only 'x' or only 'y' in it. In the addition, or also called the elimination method, our goal is to have one of the variables match in coefficient and opposite sign so that if the equations are added up, the variable will cancel.

Note: A, B, C, D, E, and F are just numbers.

Steps for addition method

1. Write both equations in standard form. If any coefficients are fractions, clear them
2. Pick one variable and multiply either/both equations by a constant to make those variable have the same coefficient with opposite signs
3. Add the equations resulting from Step 2 to eliminate onevariable
4. Solve for the remaining variable
5. Substitute the solution from Step 4 into one of the original equations to solve for the other variable
6. Write the solution as an ordered pair
7. Check that the ordered pair is a solution to both the original equations

Point: To **clear fractions,** we multiply the equation by the least common denominator. If both equations have fractions, we repeat this process for the other equation. Be sure when clearing that you multiply every term on both the left and right by the least common denominator.

Example A) Solve the system by the addition method: $3x + y = 5$ (1)
$2x - y = 0$ (2)

To solve a system of equations by addition, we start with both equations in standard form. Then we decide which variable will be easiest to eliminate. We want to have the coefficients of one variable be opposites, so that we can add the equations together and eliminate that variable. Here it is easy to eliminate 'y', since it has 1 and -1 as coefficients:

$3x + y = 5$ (1)
$2x - y = 0$ (2) Add (1) and (2)
The y's eliminate and we have one equation with one variable, 'x'.
$5x = 5$ Divide both sides by 5
$x = 1$

Substitute 'x' to the easier equation (2) to find 'y':

$2(1) - y = 0$
$2 - y = 0$
$y = 2$

So the solution is (1, 2)

Example B) Solve the system by the addition method: $2x + y = 7$
$x - 2y = 6$

Step 1. Write both equations in standard form. If any coefficients are fractions, clear them.	Both equations are in standard form, $Ax + By = C$. There are no fractions.	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$
Step 2. Make the coefficients of one variable opposites. Decide which variable you will eliminate. Multiply one or both equations so that the coefficients of that variable are opposites.	We can eliminate the y's by multiplying the first equation by 2. Multiply both sides of $2x + y = 7$ by 2.	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$ $\begin{cases} 2(2x + y) = 2(7) \\ x - 2y = 6 \end{cases}$
Step 3. Add the equations resulting from Step 2 to eliminate one variable.	We add the x's, y's, and constants.	$\begin{cases} 4x + 2y = 14 \\ x - 2y = 6 \end{cases}$ $5x \quad = 20$
Step 4. Solve for the remaining variable.	Solve for x.	$x = 4$
Step 5. Substitute the solution from Step 4 into one of the original equations. Then solve for the other variable.	Substitute $x = 4$ into the second equation, $x - 2y = 6$. Then solve for y.	$x - 2y = 6$ $4 - 2y = 6$ $-2y = 2$ $y = -1$
Step 6. Write the solution as an ordered pair.	Write it as (x, y).	$(4, -1)$
Step 7. Check that the ordered pair is a solution to **both** original equations.	Substitute $(4, -1)$ into $2x + y = 7$ and $x - 2y = 6$ Do they make both equations true? Yes!	$2x + y = 7$ $x - 2y = 6$ $2(4) + (-1) \stackrel{?}{=} 7$ $4 - 2(-1) \stackrel{?}{=} 6$ $7 = 7$ ✓ $6 = 6$ ✓ The solution is $(4, -1)$.

Example C) Note: addition and elimination are the same methods.

Solve the system by elimination: $\begin{cases} 4x - 3y = 9 \\ 7x + 2y = -6 \end{cases}$.

Solution

In this example, we cannot multiply just one equation by any constant to get opposite coefficients. So we will strategically multiply both equations by different constants to get the opposites.

	$\begin{cases} 4x - 3y = 9 \\ 7x + 2y = -6 \end{cases}$
Both equations are in standard form. To get opposite coefficients of *y*, we will multiply the first equation by 2 and the second equation by 3.	$\begin{cases} 2(4x - 3y) = 2(9) \\ 3(7x + 2y) = 3(-6) \end{cases}$
Simplify.	$\begin{cases} 8x - 6y = 18 \\ 21x + 6y = -18 \end{cases}$
Add the two equations to eliminate *y*.	$\begin{array}{r} 8x - 6y = 18 \\ 21x + 6y = -18 \\ \hline 29x \quad = 0 \end{array}$
Solve for *x*.	$x = 0$ $7x + 2y = -6$
Substitute $x = 0$ into one of the original equations.	$7 \cdot 0 + 2y = -6$
Solve for *y*.	$2y = -6$ $y = -3$
Write the solution as an ordered pair.	The ordered pair is $(0, -3)$.
Check that the ordered pair is a solution to **both** original equations. $4x - 3y = 9$, $4(0) - 3(-3) \stackrel{?}{=} 9$, $9 = 9$ ✓ $7x + 2y = -6$, $7(0) + 2(-3) \stackrel{?}{=} -6$, $-6 = -6$ ✓	
	The solution is $(0, -3)$.

Special cases: <u>no solution</u> or <u>infinitely many solutions</u>

- The system has **no solution**
  - variable from will cancel out, leaving just numerical values
  - **the numbers will not equal one another**, making a false statement
- The system has **infinitely many solution**
  - variable from will cancel out, leaving just numerical values
  - **the numbers will be equal one another**, making a true statement

Note: **<u>Fractional coefficients</u>:** clear the fractions first.

Example D) Solve the system by the addition method: $3x + 4y = 12$
$y = 3 - \frac{3}{4}x$

Solve the system by elimination: $\begin{cases} 3x + 4y = 12 \\ y = 3 - \frac{3}{4}x \end{cases}$.

Solution

	$\begin{cases} 3x + 4y = 12 \\ y = 3 - \frac{3}{4}x \end{cases}$
Write the second equation in standard form.	$\begin{cases} 3x + 4y = 12 \\ \frac{3}{4}x + y = 3 \end{cases}$
Clear the fractions by multiplying the second equation by 4.	$\begin{cases} 3x + 4y = 12 \\ 4\left(\frac{3}{4}x + y\right) = 4(3) \end{cases}$
Simplify.	$\begin{cases} 3x + 4y = 12 \\ 3x + 4y = 12 \end{cases}$
To eliminate a variable, we multiply the second equation by −1. Simplify and add.	$\begin{cases} 3x + 4y = 12 \\ \underline{-3x - 4y = -12} \end{cases}$ $0 = 0$

This is a true statement. The equations are consistent but dependent. Their graphs would be the same line. The system has infinitely many solutions.

After we cleared the fractions in the second equation, did you notice that the two equations were the same? That means we have coincident lines.

Worksheet: 4.3 Solving Systems by Addition

Find the solutions to the systems:

1. $3x - 2y = -1$

   $x + 2y = 5$

2. $3x - y = -16$

   $-2x + y = 11$

3. $x + 3y = -11$

   $-3x - y = 9$

4. $5x - 3y = -6$

   $-4x - 12y = -4$

5. $-\frac{2}{3}x + \frac{1}{2}y = -5$

   $2x - 3y = 24$

6. $-\frac{4}{3}x + \frac{3}{2}y = -7$

   $-\frac{1}{2}x + \frac{4}{5}y = -5$

7. The sum of two numbers is 39. Their difference is 9. Find the numbers.

8. The sum of two numbers is -15. Their difference is -35. Find the numbers.

4.5 Application: Value Problems

Learning Objectives: In this section, you will:

- Solve value problems by setting up a system of equations

One type of problem systems can solve for us is value problems. These are characterized by an amount of an item and the item having a value. Think about three quarters. You have three of this item that has a value of $0.25, giving a total amount of $0.75

Point: Multiply how many of an item you have by its worth to get the value. The equations we use relates to the value. It is often helpful to set up a chart.

	Number	Value	Total
Item 1			
Item 2			
Total			

We fill in the information in the chart based on the problem and use it to make a system of equations for us to work with. The total going horizontally is from the number times the value. Keep in mind not all sections will be filled in.

Point: The value column is usually empty in the total spot, as it has no meaning to our problems. When it does have a meaning, we are looking at mixture problems. As an example, if we were interested in how many quarters q and dimes d someone has, our base chart could look like this.

	Number	Value	Total
Dime	d	0.10	0.10d
Quarter	q	0.25	0.25q
Total			

If we knew the total amount of coins, we could fill in the last entry in the number column. If we need the amount of money, we could fill in the last entry in the total column. The quarter and dime having a combined value of $0.35 will not help our problem of figuring out how many of each coin we have, so we leave the value total, the last entry in the value column, blank.
Points: If the problem is about interest, remember that yearly interest is principal times interest rate.

Account	Principal	Rate	Interest
Account 1			
Account 2			
Total			

Example A) Natasha has a bank full of nickels and dimes. The total value in her bank is \$8.10. The number of dimes is 9 less than twice the number of nickels. How many nickels and how many dimes does Natasha have?

Solution:

We want the amount of nickels and dimes Natasha has, so we will use *n* for nickels and *d* for dimes. Since we know the values of each of these, we can fill in what we know so far into our chart.

	Number	Value	Total
Dimes	d	0.10	0.10d
Nickels	n	0.05	0.05n
Total			8.10

This is all the information we have to fill into the chart. This does give us one equation that relates the total worth of \$8.10 to the total value of each coin.

$$0.10d + 0.05n = 8.10$$

We know for two unknowns; we need two equations. The other equation comes from the amounts they compared in the problems. They told us the dimes are so many compared to the nickels, so we will translate that statement.

$$d = 2n - 9$$

We can solve this system by any method we like, but since the second equation is solved for *d* already, we will use substitution

$0.10(2n-9)+0.05n=8.10$	Substitution
$0.2n-0.9+0.05n=8.10$	Distribute
$0.25n-0.9=8.10$	Collect like terms
$0.25n=9$	Isolate the variable
$n=36$	Divide away the coefficient

Now that we know how many nickels Natasha has, we can substitute it to find the number of dimes.

$$d = 2n - 9 = 2(36) - 9 = 63$$

This means for our solution; Natasha has 63 dimes and 36 nickels.

Example B) The box office at a movie theater sold 147 tickets for the evening show, and receipts totaled $1302. How many $11 adult and how many $8 child tickets were sold?

Solution:

We want the amount of ticket sales for adults and children. We will let a represent adults and c represent children. To fill in our chart, we know the value of each, and we also know total ticket sales and total value.

	Number	Value	Total
Child	c	8	8c
Adult	a	11	11a
Total	147		1302

We should see that the number column and the total value column form equations for us. We know how many to combine to get the totals out, so we can write as follows.

$$c + a = 147$$
$$8c + 11a = 1302$$

Again, we will use substitution since both coefficients in the first equation are one.

We will solve for 'c': $c = 147 - a$

We can now substitute the expression for c into the second equation to find a.

$8(147 - a) + 11a = 1302$	Substitution
$1176 - 8a + 11a = 1302$	Distribute
$1176 + 3a = 1302$	Collect like terms
$3a = 126$	Isolate the variable
$a = 42$	Divide away the coefficient

Now that we know how adult tickets were sold, we can substitute to find the number of child tickets sold.

$$c = 147 - a = 147 - 42 = 105$$

This means for our solution, there were 42 adult tickets sold and 105 child tickets sold.

Worksheet: 4.5 Application: Value Problems

Solve the value problems:

1) The ticket office at the zoo sold 553 tickets one day. The receipts totaled \$3963. How many \$9 adult tickets and how many \$6 child tickets were sold?

2) Matilda has a handful of quarters and dimes, with a total value of \$8.55. The number of quarters is 3 more than twice the number of dimes. How many dimes and how many quarters does she have?

3) Adnan has \$40,000 to invest and hopes to earn 7.1% interest per year. He will put some of the money into a stock fund that earns 8% per year and the rest into bonds that earns 3% per year. How much money should he put into each fund?

4) A cashier has 30 bills, all of which are \$10 or \$20 bills. The total value of the money is \$460. How many of each type of bill does the cashier have?

5) A trust fund worth \$25,000 is invested in two different portfolios. This year, one portfolio is expected to earn 5.25% interest and the other is expected to earn 4%. Plans are for the total interest on the fund to be \$1150 in one year. How much money should be invested at each rate?

4.6 Application: Mixture Problems

Learning Objectives: In this section, you will:

- Solve mixture problems by setting up a system of equations

One type of problem systems can solve for us is mixture problems. These are characterized by combining amounts of ingredients together to get a well-mixed solution out. An example would be pouring several juices together. The result is a mixture whose concentration of any flavor depends on how much of each juice went in and the amount of flavor in each.

Point: Multiply the amount of each item by the concentration, or part, that item contains that we want to measure. This product gives us the total amount present in each item.

	Amount	Part	Total
Item 1			
Item 2			
Total			

We fill in the information in the chart based on the problem and use it to make a system of equations for us to work with. The total column at the end is for our product of amount time's part and is a measure of how much of what we care about the item contains. Keep in mind not all sections will be filled in.

Point: The last entry in the part column will have a value. This is one of the main differences to notice with value problems. Since we have a resulting solution of some sort, we have a concentration, or part, to mark. As an example, if we were mixing 3 gallons of a 10% salt solution with 2 gallons of a 15% salt solution to get a 12% salt solution, our table would look like the following.

	Amount	Concentration	Total
Item 1	3	0.10	0.3
Item 2	2	0.15	0.3
Total	5	0.12	0.6

We see that the amount column has a total of the number of gallons of our mixture. We see the rows for each item have a total of the amount multiplied by the part. The total column combines the totals from each item, so we know how much salt is in the mixture.

Point: We write the part, or concentration, as a decimal. It is a percentage in the problem, but a decimal in calculation.

Example A) John is making a large batch of chili. He needs to get a combined total of 20 pounds between the meat and beans. If John has budgeted himself \$3 per pound for his chili, how many pounds of meat and beans should he buy if meat is \$5 a pound and beans are \$ a pound?

Solution:
We want the amount, in pounds, of meat and beans John has, so we will use m for meat and b for beans. With this information, and the cost of each, we can fill in our chart.

	Amount	Part	Total
Meat	m	5	5m
Beans	b	1	b
Total	20	3	60

To solve a problem with two unknowns, we need two equations. We look to the totals to make our equations. We see that the amount column can give us one equation, where the total column can give us another.

$m + b = 20$ Amount of each item

$5m + b = 60$ Mixture total

We know for two unknowns; we need two equations. The other equation comes from the amounts they compared in the problems. They told us the dimes are so many compared to the nickels, so we will translate that statement.

$$d = 2n - 9$$

We can solve this system by any method we like, but since the second equation is solved for d already, we will use substitution

$0.10(2n - 9) + 0.05n = 8.10$	Substitution
$0.2n - 0.9 + 0.05n = 8.10$	Distribute
$0.25n - 0.9 = 8.10$	Collect like terms
$0.25n = 9$	Isolate the variable
$n = 36$	Divide away the coefficient

Now that we know how many nickels Natasha has, we can substitute it to find the number of dimes.

$d = 2n - 9 = 2(36) - 9 = 63$

This means for our solution; Natasha has 63 dimes and 36 nickels.

Example B) A 90% antifreeze solution is to be mixed with a 75% antifreeze solution to get 360 liters of an 85% solution. How many liters of the 90% and how many liters of the 75% solutions will be used?

Solution:

In this problem we are mixing these two solutions together, the 90% and the 75%, in order to make this 85% solution. We do not know the amounts of either solution going in, so we will assign them to x and y. Here is what we know

Type	Number	Concentration	Amount
90%	x	0.90	0.90x
75%	y	0.75	0.75y
85%	360	0.85	306

Notice that the first two rows are the solutions going into the pot, and the third row is the mixture we get out. Just as we have seen before, the columns make our equations. We know the numbers we are discussing and the amounts. This gives us our system.

$$x + y = 360$$
$$0.90x + 0.75y = 306$$

Again, we will use substitution since both coefficients in the first equation are one.

We will solve for x. $\quad x = 360 - y$

We can now substitute the expression for x into the second equation to find y.

$$\begin{aligned} 0.90(360 - y) + 0.75y &= 306 && \text{Substitution} \\ 324 - 0.90y + 0.75y &= 306 && \text{Distribute} \\ 324 - 0.15y &= 306 && \text{Collect like terms} \\ -0.15y &= -18 && \text{Isolate the variable} \\ y &= 120 && \text{Divide away the coefficient} \end{aligned}$$

Now that we know how much of the 75% solution to use, we can determine the amount of 90% solution.

$$x = 360 - y = 360 - 120 = 240$$

This means for our solution; we need to use 120 liters of the 75% solution with 240 liters of the 90% solution.

Worksheet: 4.6 Application: Mixture Problems

Find the solution to the systems:

1) Carson wants to make 20 pounds of trail mix using nuts and chocolate chips. His budget requires that the trail mix costs him $7.60. per pound. Nuts cost $9.00 per pound and chocolate chips cost $2.00 per pound. How many pounds of nuts and how many pounds of chocolate chips should he use?

2) Greta wants to make 5 pounds of a nut mix using peanuts and cashews. Her budget requires the mixture to cost her $6 per pound. Peanuts are $4 per pound and cashews are $9 per pound. How many pounds of peanuts and how many pounds of cashews should she use?

3) Jotham needs 70 liters of a 50% solution of an alcohol solution. He has a 30% and an 80% solution available. How many liters of the 30% and how many liters of the 80% solutions should he mix to make the 50% solution?

4) A scientist needs 65 liters of a 15% alcohol solution. She has available a 25% and a 12% solution. How many liters of the 25% and how many liters of the 12% solutions should she mix to make the 15% solution?

5) A 40% antifreeze solution is to be mixed with a 70% antifreeze solution to get 240 liters of a 50% solution. How many liters of the 40% and how many liters of the 70% solutions will be used?

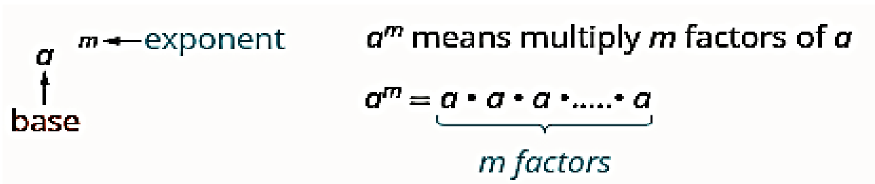
5.1 Exponent Properties

Learning Objectives: In this section, you will:

- Use exponents
- Use combinations of the rules for exponents
- Apply the rules for exponents in a geometry application.

Exponents: In the expression 4^2, the number 4 is the base and 2 is the exponent and called an exponential expression.

Exponential expression (notation)



In the expression a^m , the exponent 'm' tells us how many times we use the base a as a factor.

A **monomial** in one variable is a term of the form $\mathbf{ax^m}$, where 'a' is a constant and 'm' is a whole number.

Example A) Evaluate:

2^3 = (2)(2)(2) ------ 3 factors = 4(2) = 8

$(-3)^4$ = (-3)(-3)(-3)(-3) = 9(-3)(-3) = -27(-3) = 81 (Order of operations, multiply left to right)

-2^2 = -1(2)(2) = - 4 (Order of operations, exponent first)

$(-2)^2$ = (-2)(-2) = 4

$-(-2)^2$ = -(-2)(-2) = - 4 (Order of operations, exponent first)

Product Rule of Exponents: $a^m a^n = a^{m+n}$

When multiplying like bases, keep the base and add the powers.

Example B) Simplify:

$3^2 \cdot 3^6 \cdot 3$ Same base, add the exponents $2+6+1$

3^9 Our Solution

Example C) Simplify:

$2x^3y^5z \cdot 5xy^2z^3$ Multiply $2 \cdot 5$, add exponents on x, y and z

$10x^4y^7z^4$ Our Solution

Example D) Simplify:

$x^3 \cdot x^8 = x^{11}$ $\quad$ $2^4 \cdot 2^2 = 2^6$ $\quad$ $(x^2y)(x^3y^4) = x^5y^5$

Quotient Rule of Exponents: $\dfrac{a^m}{a^n} = a^{m-n}$

When dividing with like bases, keep the base and subtract the powers.
Note: it is always the numerator's power minus the denominator's power, see negative exponent rule later.

Example E) Simplify:

$$\frac{7^{13}}{7^5} \quad \text{Same base, subtract the exponents}$$
$$7^8 \quad \text{Our Solution}$$

Example F) Simplify:

$$\frac{5a^3b^5c^2}{2ab^3c} \quad \text{Subtract exponents on } a, b \text{ and } c$$

$$\frac{5}{2}a^2b^2c \quad \text{Our Solution}$$

Example G) Simplify:

$\frac{x^5}{x^2} = x^3$ $\qquad$ $\frac{3^5}{3^3} = 3^2$ $\qquad$ $\frac{x^2y^5}{xy^3} = xy^2$

Power of a Power Rule of Exponents: $(a^m)^n = a^{mn}$

When taking a monomial to a power, keep the base and multiply the powers.

Example H) Simplify: We can solve this two different ways:

$(a^4)^2 = (a)^4(a)^4 = a^8$ if we use exponent and Product rule (add exponents) OR
$(a^4)^2 = a^8$ quicker if we use Power rule (multiply exponents)

Example I) Simplify:

$$(x^3yz^2)^4 \quad \text{Put the exponent of 4 on each factor, multiplying powers}$$
$$x^{12}y^4z^8 \quad \text{Our solution}$$

Example J) Simplify:

$$(4x^2y^5)^3 \quad \text{Put the exponent of 3 on each factor, multiplying powers}$$
$$4^3x^6y^{15} \quad \text{Evaluate } 4^3$$
$$64x^6y^{15} \quad \text{Our Solution}$$

Example K) Simplify:

a) $(5xy)^3 = 5^3x^3y^3 = 125\,x^3y^3$

b) $(-3a^2)^3 = (-3)^3a^6 = (-3)(-3)(-3)a^6 = -27a^6$

Power of a Quotient Rule of Exponents: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

When taking a fraction to a power, raise numerator and denominator to that power.

Example L) Simplify:

$$\left(\frac{x^2}{y}\right)^4 = \frac{(x^2)^4}{y^4} = \frac{x^8}{y^4}$$

Raise numerator and denominator to 4th power, multiply exponents.

Example M) Simplify:

$\left(\frac{a^3b}{c^8d^5}\right)^2$ Put the exponent of 2 on each factor, multiplying powers

$\frac{a^6b^2}{c^8d^{10}}$ Our Solution

Example N) Simplify:

$$\left(\frac{x^2}{y}\right)^4 = \frac{(x^2)^4}{y^4} = \frac{x^8}{y^4} \qquad \left(\frac{2x}{3y^2}\right)^3 = \frac{(2x)^3}{(3y^2)^3} = \frac{2^3x^3}{3^3(y^2)^3} = \frac{8x^3}{27y^6}$$

Rules of Exponents

Product Rule of Exponents	$a^m a^n = a^{m+n}$
Quotient Rule of Exponents	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Rule of Exponents	$(a^m)^n = a^{mn}$
Power of a Product Rule of Exponents	$(ab)^m = a^m b^m$
Power of a Quotient Rule of Exponents	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Combination of rules:

Example O) Simplify:

$7a^3(2a^4)^3$ Parenthesis are already simplified, next use power rules

$7a^3(8a^{12})$ Using product rule, add exponents and multiply numbers

$56a^{15}$ Our Solution

Example P) Simplify:

$\frac{3m^8n^{12}}{(m^2n^3)^3}$ Use power rule in denominator

$\frac{3m^8n^{12}}{m^6n^9}$ Use quotient rule

$3m^2n^3$ Our solution

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Worksheet: 5.1 Exponent Properties

Simplify each of the following.

1. $a \cdot a^2 \cdot a^3$
2. $(2a^2b)(4ab^2)$
3. $(6x^2)(-3x^5)$
4. $2^3 \cdot 2^4 \cdot 2^7 \cdot 2$
5. $(3x^3)(-2x^2)$

6. $2x^3 \cdot 2x^2$
7. $\frac{x^3}{x}$
8. $\frac{18c^3}{-3c^2}$
9. $\frac{9a^3b^5}{3ab^2}$
10. $\frac{-48c^2d^4}{-8cd}$

11. $\frac{22y^6y^8}{2y^7}$
12. $(5x^2y^4)^3$
13. $(6x^4y^6)^3(xy^2)$
14. $(7xy)^2(x^2y)^3$
15. $(4x^3y^3)^3$

16. $x^2 \cdot x^7$
17. $\left(x^2\right)^7$
18. $\left(-2x^4\right)^5$
19. $6x^5 \cdot 3x^5 \cdot x$
20. $\left(3st^{12}\right)^3$

21. $\left(\frac{3m^2n^7}{m}\right)^5$
22. $\left(\frac{x^7}{y^3}\right)^4$
23. $\left(\frac{x^2y^5}{xy^2}\right)^5$
24. $\left(\frac{3^5 4^5}{3^4 4^2}\right)^3$
25. $(-3x)(3x)$

==

5.2 Negative Exponent

Learning Objectives: In this section, you will:

- Use ‘0’ as an exponent
- Use negative numbers as exponents
- Apply the rules for exponents in a geometry application.

There are a few special exponent properties that deal with exponents that are not positive.

Zero Power Rule of Exponents: $a^0 = 1$

Any number or expression raised to the zero power will always be 1.

Example A-F) Evaluate:

A) $y^0 = 1$ B) $6^0 = 1$ C) $(7a^2b)^0 = 1$

D) $-6^0 = -1$ E) $-(-6)^0 = -1$ F) $2^0 + 3^0 = 1 + 1 = 2$

Rules of Negative Exponets: $a^{-m} = \frac{1}{m}$ $\frac{1}{a^{-m}} = a^m$ $\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$

Negative exponents in the numerator must be moved to the denominator, likewise, negative exponents in the denominator need to be moved to the numerator.

When the base with negative exponent moves, the exponent becomes positive.

Example G) Simplify:

$x^{-3} = \frac{1}{x^3}$ $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ $-4x^5y^{-2} = \frac{-4x^5}{y^2}$

Example H) Simplify:

$\left(\frac{x^2}{y}\right)^{-3} = \left(\frac{y}{x^2}\right)^3 = \frac{y^3}{x^6}$ $(3x^{-2}y)(-2xy^{-3}) = -6x^{-1}y^{-2} = \frac{-6}{xy^2}$

****** All final answers should be written with positive powers.******

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Worksheet: 5.2 Negative Exponent

1. -5^0 2. $(-5)^0$ 3. $-(-2^0)$ 4. $(3x)^0$ 5. $4^0 - 6^0$

6. $(-2^0) + 2^0$ 7. $-4^0 + 6^0$ 8. $\frac{z^8}{z^3z^{-7}}$ 9. $\frac{a^{-3}b^5}{a^4b^2}$ 10. $(-2x^{-2})^3$

11. 5^{-2} 12. $2^{-1} + 3^{-1}$ 13. $\frac{a^{-3}a^5}{a^{-4}a^2}$ 14. $(-4)^{-3}$ 15. $\left(\frac{x^{-2}}{x^{-5}}\right)^{-2}$

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5.3 Scientific Notations

Learning Objectives: In this section, you will:

- Express numbers in scientific notation.
- Convert numbers in scientific notation to standard notation.
- Use scientific notation in calculations.

A number is written in **scientific notation** when it is expressed in the form

$a \times 10^n$, where $1 \le |a| < 10$ and n is an integer.

A number in scientific notation is always written with the decimal point after the first nonzero digit and then multiplied by the appropriate power of 10.

To write long numbers, it is typical to use scientific notation, a system based on the powers of 10.

10^0	= 1			
10^1	= 10		10^{-1}	= .1
10^2	= 100		10^{-2}	= .01
10^3	= 1000	in the same way	10^{-3}	= .001
10^4	= 10000		10^{-4}	= .0001

Converting Decimal to Scientific Notation

Example A) Write 435,000 in scientific notation (larger than 1 numbers)

4.25 Move decimal point after first digit (the number must be between 1 and 10)

$4.35 \text{ x } 10^5$ The exponent is determined by the number of places the decimal is moved: 5 here. We use positive exponents, since we have to multiply 4.56 by 10^5= 100,000 to get back the original number. 'x' means multiplication.

Example B) Write .000456 in scientific notation (smaller than 1 numbers)

4.56 Move decimal point after first digit (the number must be between 1 and 10)

$4.35 \text{ x } 10^{-4}$ The exponent is determined by the number of places the decimal is moved: -4 here. We use negative exponents, since we have to multiply 4.35 by 10^{-4} = .0001 to get back the original number. 'x' means multiplication.

Example C) 10,400,000 in scientific notation equals $1.04 \text{ x } 10^7$

Example D) .00204 in scientific notation equals $2.04 \text{ x } 10^{-3}$

Converting from Scientific Notation to Standard Notation (Decimal)

We can also turn a number notated scientifically into a standard notation-decimal number by reversing this process:

Example E) Convert 8.7×10^9 into a decimal.

$8.7 \times 10^9 = 8,700,000,000$ Convert this to decimal by moving the decimal point 9 places to the right (positive exponent)

Example F) Negative exponent means a number that is less than one:

Convert 5.4×10^{-7} into a decimal.

$5.4 \times 10^{-7} = .00000054$ Convert this to decimal by moving the decimal point 7 places to the left (negative exponent means we divide).

Example G) 6.3×10^4 = move the decimal point 4 places to the right = 63000

Example H) 9.32×10^{-3} = move the decimal point 3 places to the left = .00932

Operations on scientific numbers:

Example I) Multiply scientific numbers, find result in scientific notation:

$$(2.1 \times 10^{-7})(3.7 \times 10^5) \quad \text{Deal with numbers and } 10's \text{ separately}$$
$$(2.1)(3.7) = 7.77 \quad \text{Multiply numbers}$$
$$10^{-7}10^5 = 10^{-2} \quad \text{Use product rule on } 10's \text{ and add exponents}$$
$$7.77 \times 10^{-2} \quad \text{Our Solution}$$

Example J) Divide scientific numbers, find result in scientific notation (Note: this is an example when your number is not scientific notation after the division and you must change it to scientific notation firsts.)

$$\frac{2.014 \times 10^{-3}}{3.8 \times 10^{-7}} \quad \text{Deal with numbers and } 10's \text{ separately}$$
$$\frac{2.014}{3.8} = 0.53 \quad \text{Divide numbers}$$
$$0.53 = 5.3 \times 10^{-1} \quad \text{Change this number into scientific notation}$$
$$\frac{\mathbf{10^{-1}}10^{-3}}{10^{-7}} = 10^3 \quad \text{Use product and quotient rule, using } 10^{-1} \text{ from the conversion}$$
Be careful with signs:
$$(-1) + (-3) - (-7) = (-1) + (-3) + 7 = 3$$
$$5.3 \times 10^3 \quad \text{Our Solution}$$

Worksheet: 5.3 Scientific Notations

Write in scientific notation:

1. 575

2. 87,400

3. 0.643

4. 0.000802

Write in decimal notation:

5. 2.54×10^{1}

6. 6.19×10^{3}

7. 4.64×10^{-1}

8. 7×10^{-3}

Solve the following problems:

9. The distance from the earth to the nearest star outside our solar system is approximately 25,700,000,000,000. When expressed in scientific notation, what is the value of *n*.

   $2.57 \text{ x } 10^{n}$

10. One light year is approximately $5.87 \text{ x } 10^{12}$ miles. Use scientific notation to express this distance in feet (Hint: 5,280 feet = 1 mile).

11. John travels regularly for his job. In the past five years he has traveled approximately 355,000 miles. Convert his total miles into scientific notation.

Multiply/divide scientific numbers, find result in scientific notation:

12. $(7 \text{ x } 10^{-1})(2 \text{ x } 10^{-3})$

13. $(2.6 \text{ x } 10^{-2})(6 \text{ x } 10^{-2})$

14. $\dfrac{4.9 \times 10^{1}}{2.7 \times 10^{-3}}$

15. $\dfrac{5.33 \times 10^{-6}}{9.62 \times 10^{-2}}$

5.4 Introduction to Polynomials

Learning Objectives: In this section, you will:

- Identify polynomials, monomials, binomials, and trinomials
- Add and subtract monomials
- Add and subtract polynomials
- Evacuate a polynomial for a given value.

Polynomial—A monomial, or two or more monomials combined by addition or subtraction.
- **monomial** —A polynomial with exactly **one term** is called a monomial.
- **binomial** —A polynomial with exactly **two terms** is called a binomial.
- **trinomial** —A polynomial with exactly **three terms** is called a trinomial.

Example A) Some examples of polynomials:

Polynomial	$y+1$	$4a^2-7ab+2b^2$	$4x^4+x^3+8x^2-9x+1$	
Monomial	14	$8y^2$	$-9x^3y^5$	$-13a^3b^2c$
Binomial	$a+7b$	$4x^2-y^2$	y^2-16	$3p^3q-9p^2q$
Trinomial	$x^2-7x+12$	$9m^2+2mn-8n^2$	$6k^4-k^3+8k$	z^4+3z^2-1

The Degree of a Polynomial
- The **degree of a term** is the sum of the exponents of its variables.
- The **degree of a constant** is 0.
- The **degree of a polynomial** is the highest degree of all its terms.

Example B) Some examples of finding number of terms and degrees:

Polynomial	Number of terms	Type	Degree of terms	Degree of polynomial
$7y^2-5y+3$	3	Trinomial	2, 1, 0	2
$-2a^4b^2$	1	Monomial	6	6
$3x^5-4x^3-6x^2+x-8$	5	Polynomial	5, 3, 2, 1, 0	5

<u>Evaluate polynomials:</u> replace the variable with the given number value and evaluate the polynomial.

Example C) Evaluate the trinomial: $2x^2 - 4x + 6$ when $x = -4$

$$\begin{array}{rl} 2x^2-4x+6 \text{ when } x=-4 & \text{Replace variable } x \text{ with } -4 \\ 2(-4)^2-4(-4)+6 & \text{Exponents first} \\ 2(16)-4(-4)+6 & \text{Multiplication (we can do all terms at once)} \\ 32+16+6 & \text{Add} \\ 54 & \text{Our Solution} \end{array}$$

Add/subtract polynomials: we combine like terms (add/subtract coefficients and keep variable)

Example D) Add $4x^3 - 2x + 8$ and $3x^3 - 9x^2 - 11$

$$\begin{array}{rl} (4x^3-2x+8)+(3x^3-9x^2-11) & \text{Combine like terms } 4x^3+3x^3 \text{ and } 8-11 \\ 7x^3-9x^2-2x-3 & \text{Our Solution} \end{array}$$

Example E) Subtract $3x^2 + 6x - 4$ from $5x^2 - 2x + 7$

Note: remember; 'subtract from' will reverse order

$$\begin{array}{rl} (5x^2-2x+7)-(3x^2+6x-4) & \text{Distribute negative through second part} \\ 5x^2-2x+7-3x^2-6x+4 & \text{Combine like terms } 5x^2-3x^3, -2x-6x, \text{ and } 7+4 \\ 2x^2-8x+11 & \text{Our Solution} \end{array}$$

Example F) Simplify: *(2x – 5y) – (3x + 2y)*

(2x – 5y) – (3x + 2y)	*Distribute negative through second part*
<u>2x</u> – 5y <u>– 3x</u> - 2y	*Combine like terms*
<u>2x - 3x</u> – 5y - 2y	*Combine like terms*
<u>(2-3)x</u> +(-5-2)y	
-x -7y	Our solution

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Worksheet: 5.4 Introductions to Polynomials

1) Evaluate: For the polynomial 5x2 − 8x + 4, find the value when:

a) $x = 4$
b) $x = -2$
c) $x = 0$

2) Determine whether each polynomial is a monomial, binomial, trinomial, or other polynomial. Then, find the degree of each polynomial.

a) -5
b) $8y^3-7y^2-y-3$
c) $-3x^2y-5xy+9xy^3$
d) $81m^2-4n^2$
e) $-3x^6y^3z$

3) Add or subtract:

a) $25y^2+15y^2$
b) $16pq^3-(-7pq^3)$

4) Find the sum: $\left(7y^2-2y+9\right)+\left(4y^2-8y-7\right)$

5) Add or subtract:

a) $\left(7m^2+mn-8n^2\right)+\left(3m^2+2mn\right)$
b) $\left(a^2-b^2\right)-\left(a^2+3ab-4b^2\right)$
c) $\left(p^3-3p^2q\right)+\left(2pq^2+4q^3\right)-\left(3p^2q+pq^2\right)$

6) Subtract: $(4m^2-6m-3)-(2m^2+m-7)$

7) Subtract $\left(9x^2+2\right)$ from $\left(12x^2-x+6\right)$

8) Find the difference of $\left(z^2-3z-18\right)$ and $\left(z^2+5z-20\right)$

==

5.5 Multiply Polynomials

Learning Objectives: In this section, you will:

- Multiply a monomial and a polynomial
- Multiply two polynomials
- Multiply binomials by the Foil method

Multiply a polynomial by a monomial:
- **use the distributive property**
- **multiply coefficients (numbers in front of variables)**
- **add exponents of like variables**

Example A) Multiply: $-2y(4y^2+3y-5)$

	$-2y\,(4y^2+3y-5)$
Distribute.	$-2y \cdot 4y^2 + (-2y) \cdot 3y - (-2y) \cdot 5$
Multiply.	$-8y^3 - 6y^2 + 10y$

Multiply a binomial by a binomial:

Two methods: distribute or FOIL

Example B) Multiply using the distributive property and FOIL Method: $(x + 3)(x + 5)$

FOIL stand for 'First, Outer, Inner, Last'

Distributive Property	FOIL
$(x+3)(x+7)$	$(x+3)(x+7)$
$x(x+7)+3(x+7)$	
$\underset{F}{x^2}+\underset{O}{7x}+\underset{I}{3x}+\underset{L}{21}$	$\underset{F}{x^2}+\underset{O}{7x}+\underset{I}{3x}+\underset{L}{21}$
$x^2+10x+21$	$x^2+10x+21$

Example C) Multiply using distributive property: $(4y + 3)(2y - 5)$

Distribute.	$4y(2y-5)+3(2y-5)$
Distribute again.	$8y^2-20y+6y-15$
Combine like terms.	$8y^2-14y-15$

HOW TO

Multiply two binomials using the FOIL method

Step 1. Multiply the *First* terms.

Step 2. Multiply the *Outer* terms.

Step 3. Multiply the *Inner* terms.

Step 4. Multiply the *Last* terms.

Step 5. Combine like terms, when possible.

$$(\overset{first}{a} + \overset{last}{b})(\overset{first}{c} + \overset{last}{d})$$

inner: b, c — outer: a, d

Say it as you multiply!
FOIL
First
Outer
Inner
Last

Example D) Multiply using FOIL method: $(n^2 + 4)(n-1)$

	$(n^2+4)(n-1)$
Multiply the *First*.	$\underset{F}{n^3} + \underset{O}{\_\_} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Outer*.	$\underset{F}{n^3} \underset{O}{-n^2} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Inner*.	$\underset{F}{n^3} \underset{O}{-n^2} + \underset{I}{4n} + \underset{L}{\_\_}$
Multiply the *Last*.	$\underset{F}{n^3} \underset{O}{-n^2} + \underset{I}{4n} \underset{L}{-4}$
Combine like terms—there are none.	n^3-n^2+4n-4

Multiply a polynomial by a polynomial: use distributive property

Example E) Multiply: $(b + 3)(2b^2 - 5b + 8)$

Distribute.	$b(2b^2 - 5b + 8) + 3(2b^2 - 5b + 8)$
Multiply.	$2b^3 - 5b^2 + 8b + 6b^2 - 15b + 24$
Combine like terms.	$2b^3 + b^2 - 7b + 24$

Example F) Multiply:

$$\begin{array}{rl}
(2x-5)(4x^2-7x+3) & \text{Distribute } 2x \text{ and } -5 \\
(2x)(4x^2)+(2x)(-7x)+(2x)(3)-5(4x^2)-5(-7x)-5(3) & \text{Multiply out each term} \\
8x^3-14x^2+6x-20x^2+35x-15 & \text{Combine like terms} \\
8x^3-34x^2+41x-15 & \text{Our Solution}
\end{array}$$

Example G) Use three methods to multiply binomials:

$(4x - 5y)(2x - y)$

Distribute

$$\begin{array}{c}
4x(2x-y)-5y(2x-y) \\
8x^2-4xy-10xy-5y^2 \\
8x^2-14xy-5y^2
\end{array}$$

FOIL

$$\begin{array}{c}
2x(4x)+2x(-5y)-y(4x)-y(-5y) \\
8x^2-10xy-4xy+5y^2 \\
8x^2-14xy+5y^2
\end{array}$$

Rows

$$\begin{array}{r}
2x-y \\
\underline{\times\ 4x-5y} \\
-10xy+5y^2 \\
\underline{8x^2-4xy} \\
8x^2-14xy+5y^2
\end{array}$$

==

Worksheet: 5.5 Multiply Polynomials

Find each product:

1. $-6(p - 7)$
2. $4k(8k + 4)$
3. $-3n^2 (6n + 7)$
4. $(n + 6)(n + 8)$
5. $(b + 3)(b - 5)$
6. $(r - 8)(4r + 8)$
7. $(7n - 6)(7n + 6)$
8. $(5x + y)(5x - 2y)$
9. $(r - 7)(6r^2 - r + 5)$
10. $(6n - 4)(2n^2 - 2n + 5)$
11. $3(3x - 4)(2x + 1)$
12. $-7(x - 5)(x - 2)$
13. $(x - 2)(x + 3)(x - 4)$
14. Find the formulas for the perimeter and area of a rectangle where the width = 2x-1 and the length = 3x + 2
15. Find the formulas for the perimeter and area of a rectangle in terms of ‘w’ where the length is 2 more than the width (‘w’).
16. Find the formulas for the perimeter and area of a rectangle in terms of ‘w’ where the length is 3 less than twice the width (‘w’).

==

5.6 Multiply Special Products

Learning Objectives: In this section, you will:

- Square binomials.
- Find the product of the sum and difference of two terms.
- Find greater powers of binomials.

Squaring a Binomial

Let's start by looking at $(x+9)^2$.	
What does this mean?	$(x+9)^2$
It means to multiply $(x+9)$ by itself.	$(x+9)(x+9)$
Then, using FOIL, we get:	$x^2+9x+9x+81$
Combining like terms gives:	$x^2+18x+81$

Square of a Binomial

The square of a binomial is a trinomial consisting of

the square of the first term + twice the product of the two terms + the square of the last term.

For x and y, the following hold.

$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$

Example A) Square the binomial $(a + b)^2$ using distributive property

$(a+b)^2$	Squared is same as multiplying by itself
$(a+b)(a+b)$	Distribute $(a+b)$
$a(a+b)+b(a+b)$	Distribute again through final parenthesis
$a^2+ab+ab+b^2$	Combine like terms $ab+ab$
$a^2+2ab+b^2$	Our Solution

Example B) Square the binomial $(x - 5)^2$ using perfect square formula above

$(x-5)^2$	Recognize perfect square
x^2	Square the first
$2(x)(-5)=-10x$	Twice the product
$(-5)^2=25$	Square the last
$x^2-10x+25$	Our Solution

Example C) Square the binomial $(2x + 5)^2$ using perfect square formula

$$\begin{aligned} (2x+5)^2 &\quad \text{Recognize perfect square} \\ (2x)^2 = 4x^2 &\quad \text{Square the first} \\ 2(2x)(5) = 20x &\quad \text{Twice the product} \\ 5^2 = 25 &\quad \text{Square the last} \\ 4x^2 + 20x + 25 &\quad \text{Our Solution} \end{aligned}$$

A **conjugate pair** is two binomials of the form **(a – b) and (a + b)**

Product of conjugates (or difference of squares)

$$(a-b)(a+b) = a^2 - b^2$$

$$\underbrace{(a-b)(a+b)}_{\text{conjugates}} = \underbrace{a^2 \overset{\text{difference}}{-} b^2}_{\text{squares}}$$

The product is called a difference of squares.
To multiply conjugates, square the first term, square the last term, write it as a difference of squares.

Example D) Multiply $(a + b)(a – b)$

$$\begin{aligned} (a+b)(a-b) &\quad \text{Distribute } (a+b) \\ a(a+b) - b(a+b) &\quad \text{Distribute } a \text{ and } -b \\ a^2 + ab - ab - b^2 &\quad \text{Combine like terms } ab - ab \\ a^2 - b^2 &\quad \text{Our Solution} \end{aligned}$$

Example E) Multiply $(x - 5)(x + 5)$

$$\begin{aligned} (x-5)(x+5) &\quad \text{Recognize sum and difference} \\ x^2 - 25 &\quad \text{Square both, put subtraction between. Our Solution} \end{aligned}$$

Example F) Multiply $(2x – 6y)(2x + 6y)$

$$\begin{aligned} (2x-6y)(2x+6y) &\quad \text{Recognize sum and difference} \\ 4x^2 - 36y^2 &\quad \text{Square both, put subtraction between. Our Solution} \end{aligned}$$

Example G) Review the difference between the three problems:

$(4x-7)(4x+7)$	$(4x+7)^2$	$(4x-7)^2$
$16x^2-49$	$16x^2+56x+49$	$16x^2-56x+49$

Binomial Squares	Product of Conjugates
$(a+b)^2=a^2+2ab+b^2$	$(a-b)(a+b)=a^2-b^2$
$(a-b)^2=a^2-2ab+b^2$	
• Squaring a binomial	• Multiplying conjugates
• Product is a **trinomial**	• Product is a **binomial.**
• Inner and outer terms with FOIL are **the same.**	• Inner and outer terms with FOIL are **opposites.**
• Middle term is **double the product** of the terms	• There is **no** middle term.

==

<u>Worksheet: 5.6 Multiply Special Products</u>

Find each product:

1) $(x + 5)^2$
2) $(2x - 1)^2$
3) $(3x - 2y)^2$
4) $(x - 3y)^2$
5) $\left(x + \frac{1}{2}\right)^2$
6) $3(a - 4)^2$
7) $-5(w - y)^2$
8) $(x - 3)(x + 3)$
9) $(d + 7)(d - 7)$
10) $(3x - 1)(3x + 1)$
11) $(5x - 2y)(5x + 2y)$
12) $(c + 11)(c - 11)$
13) $\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$
14) $5(x + 1)(x - 1)$
15) $-3x(2x + 1)(2x - 1)$
16) $(x - 5)^3$
17) $(2x - 3)^3$

==

5.7 Divide Polynomials

Learning Objectives: In this section, you will:

- Divide a polynomial by a monomial.
- Divide a polynomial by a polynomial.
- Apply polynomial division in a geometry application.

Review: Divide Monomials: We'll try to rediscover the property by looking at some examples.

Consider	$\frac{x^5}{x^2}$	and	$\frac{x^2}{x^3}$
What do they mean?	$\frac{x\cdot x\cdot x\cdot x\cdot x}{x\cdot x}$		$\frac{x\cdot x}{x\cdot x\cdot x}$
Use the Equivalent Fractions Property.	$\frac{\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x}{\cancel{x}\cdot\cancel{x}}$		$\frac{\cancel{x}\cdot\cancel{x}\cdot 1}{\cancel{x}\cdot\cancel{x}\cdot x}$
Simplify.	x^3		$\frac{1}{x}$

QUOTIENT PROPERTY FOR EXPONENTS

If a is a real number, $a \neq 0$, and m and n are whole numbers, then

$$\frac{a^m}{a^n} = a^{m-n}, m > n \text{ and } \frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m$$

Example A) Divide or find the quotient of $56x^7$ and $8x^3$

	$56x^7 \div 8x^3$
Rewrite as a fraction.	$\frac{56x^7}{8x^3}$
Use fraction multiplication.	$\frac{56}{8}\cdot\frac{x^7}{x^3}$
Simplify and use the Quotient Property.	$7x^4$

Example B) Divide or find the quotient of $8a^3b^2$ and $-10a^5b$

Rewrite as a fraction $\frac{8a^3b^2}{-10a^5b}$

Use fraction multiplication $\frac{8}{-10}\cdot\frac{a^3}{a^5}\cdot\frac{b^2}{b}$

Simplify $-\frac{4b}{5a^2}$

Divide a polynomial by a monomial: divide each term in the numerator by the monomial in denominator.

Example C) Find the quotient: $(15x^3y - 35xy^2) \div (-5xy)$

	$(15x^3y - 35xy^2) \div (-5xy)$
Rewrite as a fraction.	$\frac{15x^3y-35xy^2}{-5xy}$
Separate the terms.	$\frac{15x^3y}{-5xy} - \frac{35xy^2}{-5xy}$
Simplify.	$-3x^2 + 7y$

Example D) Divide:

$\frac{9x^5 + 6x^4 - 18x^3 - 24x^2}{3x^2}$ Divide each term in the numerator by $3x^2$

$\frac{9x^5}{3x^2} + \frac{6x^4}{3x^2} - \frac{18x^3}{3x^2} - \frac{24x^2}{3x^2}$ Reduce each fraction, subtracting exponents

$3x^3 + 2x^2 - 6x - 8$ Our Solution

Example E) Divide:

$\frac{8x^3 + 4x^2 - 2x + 6}{4x^2}$ Divide each term in the numerator by $4x^2$

$\frac{8x^3}{4x^2} + \frac{4x^2}{4x^2} - \frac{2x}{4x^2} + \frac{6}{4x^2}$ Reduce each fraction, subtracting exponents

Remember negative exponents are moved to denominator

$2x + 1 - \frac{1}{2x} + \frac{3}{2x^2}$ Our Solution

Divide a polynomial by a polynomial.

To divide a polynomial by a polynomial, we follow a procedure similar to long division of numbers.

Example F) Divide (x^2 + 9x + 20) by (x + 5)

Note: realize that we cannot follow processes earlier, since divisor is not monomial.

$$(x^2+9x+20) \div (x+5)$$

Write it as a long division problem.
Be sure the dividend is in standard form.

$$x+5\overline{)x^2+9x+20}$$

Divide x^2 by x. It may help to ask yourself, "What do I need to multiply x by to get x^2?"

$$\begin{array}{r} x \\ x+5\overline{)x^2+9x+20} \end{array}$$

Put the answer, x, in the quotient over the x term.
Multiply x times $x+5$. Line up the like terms under the dividend.

$$\begin{array}{r} x \\ x+5\overline{)x^2+9x+20} \\ \underline{x^2+5x} \end{array}$$

Subtract x^2+5x from x^2+9x.
You may find it easier to change the signs and then add.
Then bring down the last term, 20.

$$\begin{array}{r} x \\ x+5\overline{)\;x^2+\;\;9x+20} \\ \underline{-x^2+(-5x)} \\ 4x+20 \end{array}$$

Divide $4x$ by x. It may help to ask yourself, "What do I need to multiply x by to get $4x$?"
Put the answer, 4, in the quotient over the constant term.

$$\begin{array}{r} x+4 \\ x+5\overline{)\;x^2+\;\;9x+20} \\ \underline{-x^2+(-5x)} \\ 4x+20 \end{array}$$

Multiply 4 times $x+5$.

$$\begin{array}{r} x+4 \\ x+5\overline{)\;x^2+\;\;9x+20} \\ \underline{-x^2+(-5x)} \\ 4x+20 \\ \underline{4x+20} \end{array}$$

Subtract $4x+20$ from $4x+20$.

$$\begin{array}{r} x+\;\;4 \\ x+5\overline{)\;x^2+\;\;9x+\;\;20} \\ \underline{-x^2+(-5x)} \\ 4x+\;\;20 \\ \underline{-4x+(-20)} \\ 0 \end{array}$$

Check:
Multiply the quotient by the divisor. $(x+4)(x+5)$

You should get the dividend. $x^2+9x+20$ ✓

Steps of Dividing Polynomials:

1. Divide front terms
2. Multiply this term by the divisor
3. Change the sign of the terms and combine
4. Bring down the next term
5. Repeat

Example G) Find the quotient: $(x^4 - x^2 + 5x - 6) \div (x + 2)$

$$(x^4 - x^2 + 5x - 6) \div (x + 2)$$

Write it as a long division problem. Be sure the dividend is in standard form with placeholders for missing terms.

$$x+2\,\overline{)\,x^4+0x^3-x^2+5x-6}$$

Divide x^4 by x.
Put the answer, x^3, in the quotient over the x^3 term.
Multiply x^3 times $x+2$. Line up the like terms.
Subtract and then bring down the next term.

$$\begin{array}{r} x^3 \\ x+2\,\overline{)\,x^4+0x^3-x^2+5x-6} \\ \underline{-(x^4+2x^3)} \\ -2x^3-x^2 \end{array}$$

It may be helpful to change the signs and add.

Divide $-2x^3$ by x.
Put the answer, $-2x^2$, in the quotient over the x^2 term.
Multiply $-2x^2$ times $x+1$. Line up the like terms
Subtract and bring down the next term.

$$\begin{array}{r} x^3-2x^2 \\ x+2\,\overline{)\,x^4+0x^3-x^2+5x-6} \\ \underline{-(x^4+2x^3)} \\ -2x^3-x^2 \\ \underline{-(-2x^3-4x^2)} \\ 3x^2+5x \end{array}$$

It may be helpful to change the signs and add.

Divide $3x^2$ by x.
Put the answer, $3x$, in the quotient over the x term.
Multiply $3x$ times $x+1$. Line up the like terms.
Subtract and bring down the next term.

$$\begin{array}{r} x^3-2x^2+3x \\ x+2\,\overline{)\,x^4+0x^3-x^2+5x-6} \\ \underline{-(x^4+2x^3)} \\ -2x^3-x^2 \\ \underline{-(-2x^3-4x^2)} \\ 3x^2+5x \\ \underline{-(3x^2+6x)} \\ -x-6 \end{array}$$

It may be helpful to change the signs and add.

Divide $-x$ by x.
Put the answer, -1, in the quotient over the constant term.
Multiply -1 times $x+1$. Line up the like terms.
Change the signs, add.

Write the remainder as a fraction with the divisor as the denominator.

$$\begin{array}{r} x^3-2x^2+3x-1-\dfrac{4}{x+2} \\ x+2\,\overline{)\,x^4+0x^3-x^2+5x-6} \\ \underline{-(x^4+2x^3)} \\ -2x^3-x^2 \\ \underline{-(-2x^3-4x^2)} \\ 3x^2+5x \\ \underline{-(3x^2+6x)} \\ -x-6 \\ \underline{-(-x-2)} \\ -4 \end{array}$$

It may be helpful to change the signs and add.

To check, multiply
$(x+2)\left(x^3-2x^2+3x-1-\dfrac{4}{x+2}\right)$.
The result should be x^4-x^2+5x-6.

Example H) Find the quotient: $(8a^3 + 27) \div (2a + 3)$

This time we will show the division all in one step.
We need to add two placeholders in order to divide.

$$(8a^3 + 27) \div (2a + 3)$$

$$
\begin{array}{r l l}
 & \,4a^2 - 6a + 9 & \\
2a+3 & \overline{)\,8a^3 + 0a^2 + 0a + 27} & \\
 & \underline{-(8a^3 + 12a^2)} & \leftarrow 4a^2(2a+3) \\
 & \qquad -12a^2 + 0a & \\
 & \qquad \underline{-(-12a^2 - 18a)} & \leftarrow 6a(2a+3) \\
 & \qquad\qquad 18a + 27 & \\
 & \qquad\qquad \underline{-(18a + 27)} & \leftarrow 9(2a+3) \\
 & \qquad\qquad\qquad 0 &
\end{array}
$$

To check, multiply $(2a + 3)\left(4a^2 - 6a + 9\right)$.

The result should be $8a^3 + 27$.

==

Worksheet: 5.7 Divide Polynomials

Divide a monomial by a monomial:

1) $20m^8n^4 \div (30m^5n^9)$

2) $\frac{45x^5y^9}{-60x^8y}$

3) $\frac{(6ab^2)(4a^3b^5)}{(12a^3b^2)(a^5b)}$

Divide a polynomial by a monomial:

4) $(9n^4 + 6n^3) \div (3n)$

5) $\frac{10a^2+5a-4}{-5a}$

6) $\frac{66x^3y^2-110x^2y^3-44x^4y^3}{11x^2y^2}$

Divide a polynomial by a polynomial (use long division):

7) $(y^2 + 7y + 12) \div (y + 3)$

8) $(a^2 - 2a - 35) \div (a + 5)$

9) $(4x^2 - 17x - 15) \div (x - 5)$

10) $(p^2 + 11p + 16) \div (p + 8)$

11) $(3b^3 + b^2 + 4) \div (b + 1)$

12) $(64y^3 - 27) \div (4y - 3)$

13) $(a^4 - 9) \div (a^2 + 3)$

===

Beginning Algebra
Answer Key for the Worksheets

0.1 Integers

1) -100
2) 20
3) -5
4) >
5) >
6) >
7) <
8) -5
9) -14
10) 2
11) -30
12) -9
13) -16
14) -6
15) 38
16) 4 degrees
17) 61°F
18) $330
19) -28
20) 40
21) -15
22) 6
23) 4
24) -2

0.2 Fractions

1) $\frac{19}{4}$
2) $\frac{23}{16}$
3) $17\frac{1}{2}$
4) $2\frac{1}{2}$
5) ½
6) $\frac{5}{6}$
7) $\frac{15}{13}$ *or* $1\frac{2}{13}$
8) $\frac{4}{3}$
9) $\frac{3}{10}$
10) 8
11) $-\frac{1}{15}$
12) 35
13) 5
14) $\frac{12}{5}$
15) -27
16) $\frac{39}{2}$
17) $\frac{36}{65}$
18) 32
19) $\frac{4}{5}$
20) $-\frac{2}{7}$
21) $\frac{11}{15}$
22) $\frac{13}{24}$
23) $-\frac{13}{42}$
24) $-\frac{23}{4}$
25) $\frac{185}{12}$
26) $\frac{3}{2}$

0.3 Order of Operations

1) 34
2) -7
3) 33
4) 102
5) 8
6) 5
7) $\frac{11}{3}$
8) $\frac{1}{8}$
9) 26
10) $-\frac{7}{5}$
11) $\frac{7}{3}$
12) $\frac{5}{2}$
13) $\frac{5}{4}$
14) $\frac{17}{8}$
15) $\frac{16}{15}$
16) $5\frac{1}{4}$

0.4 Properties of Algebra (Simplify, Evaluate, Translate Expressions)

1) 32
2) 5
3) 6
4) 34
5) 34
6) 12/7
7) -9x
8) -7x - 9
9) -m
10) 10n + 3
11) 24v + 27
12) 5 – 9a
13) -10 – 20x
14) -2n – 2
15) 10p + 1
16) 14b + 90
17) 60v – 7
18) 11x
19) x – 8
20) x + 7
21) m/n
22) x^2
23) 3a + b
24) 3(a + b)
25) $7 + x^3$
26) $\frac{1}{2}xy$
27) $2ab$
28) 2ab
29) 5x – 2
30) 2x + 3x = 5x
31) a^3+b
32) $(a+b)^3$

1.1 Solving Linear Equations - One Step Equations

1) 7
2) 11
3) -5
4) 4
5) 10
6) 6
7) -19
8) -6
9) 18
10) 6
11) -20
12) -7
13) -108
14) 5
15) -8
16) 4

1.2 Linear Equations - Two Steps Equations

1) -4
2) 7
3) -14
4) -2
5) 10
6) -12
7) 0
8) 12
9) -10
10) -16
11) -4y + 11 = -5;
    y = 4
12) 8 + 5x = 25;
    x = -14
13) 5 + 4x = 25;
    x = 5
14) .75x + 2.35 = 10;
    x = 10.2 mi
15) 3x + 150 = 300;
    x = 50 guests

1.3 General Linear Equations - Multi Steps Equations

1) b = 2, conditional equation
2) x = $\frac{2}{7}$, conditional equation
3) m = 3, conditional equation
4) y = 0, conditional equation
5) m = 3, conditional equation
6) y = -5, conditional equation
7) p = -4, conditional equation
8) all real numbers, identity
9) all real numbers, identity
10) no solution, contradiction

1.4 Solving with Fractions

1) $n = \frac{1}{6}$
2) $k = -\frac{4}{3}$
3) $n = 0$
4) $b = -2$
5) $r = 1$
6) $p = \frac{3}{4}$
7) $v = \frac{3}{2}$
8) $x = -\frac{45}{4}$
9) $x = \frac{1}{2}$
10) $x = -\frac{11}{2}$
11) $x = 13$

1.5 Formulas

1) $c = b - a$
2) $x = g + f$
3) $L = S - 2B$
4) $x = \frac{c-b}{a}$
5) $L = \frac{q+6p}{6}$
6) $h = \frac{S-\pi r^2}{\pi r}$
7) $T = \frac{R-b}{a}$
8) $r = \frac{d}{t}$
9) $w = \frac{V}{lh}$
10) $h = \frac{3V}{\pi r^2}$
11) $v = \frac{h+16t^2}{t}$
12) $k = qr + m$

1.8 Application: Number/Geometry

Number Problems

1) X=6
2) X=5
3) X=-4
4) X=32
5) X=-13
6) X=62
7) X=16
8) Son = \$20 & Mr. Brown = \$200
9) Boys = 15 & girls = 30
10) 14ft and 16ft
11) \$1644

Geometry Problems

1) The first angle 56, the second angle 56, & the third angle 68
2) The first angle 64, the second angle 64, & the third angle 52
3) The first angle 30, the second angle 120, & the third angle 30
4) The first angle 40, the second angle 80, & the third angle 60
5) W = 30, L = 45
6) W = 56, L = 96
7) W = 57, L = 83
8) W = 17, L = 31
9) W = 112, L = 192

1.9 Other Applications: Age, Sales Tax, Discount, and Commission Problems

1) Boy = 16 & brother = 6
2) Son = 10 & father = 40
3) (a) The sales tax is \$20.50 & (b) the total cost is \$270.50 (b)
4) 9%
5) 7.5%
6) \$ 273
7) \$394.20
8) 6%
9) 4%
10) \$450
11) (a) \$11.60, (b) \$17.40
12) (a) \$ 256.75 (b) \$138.25

3.1 Solve and Graph Inequalities

1) $(-5, \infty)$
2) $(4, \infty)$
3) $(-\infty, -2]$
4) $(-\infty, 1]$
5) $(-\infty, 5]$
6) $(-5, \infty)$
7) $x < -2$
8) $x \leq 1$
9) $x \geq 5$
10) $(-5, \infty)$
11) $(-\infty, 110)$
12) $[5/2, \infty)$
13) $[-1, \infty)$
14) $(3/2, \infty)$
15) $(3, \infty)$
16) $(-\infty, 20/3)$
17) $[-18, \infty)$

2.1 Graphing: Points and Lines

Plot Points

1) B
2) D
3) O
4) H
5) C
6) F
7) (-3, -2)
8) (1, -6)
9) (8, 0)
10) (7, 8)
11) (-8, 0)
12) (5, 5)
19) IV
20) III
21) II
22) I

Graphing

1) (0, 3); (4, -45); (-2, 27)
2) (0, 3); (2, 0); (-2, 6)

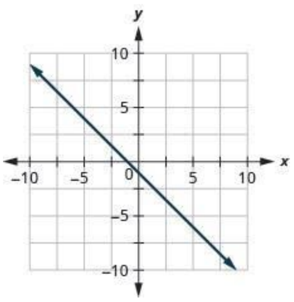
2.2 Slope

1) 3/5
2) -2/3
3) -3/5
4) 5/11
5) 1/16
6) -12/31
7) 1/16
8) x-int: (2, 0) ; y-int: (0, -6)
9) x-int: (-5, 0) ; y-int: (0, -5)
10) x-int: (20, 0) ; y-int: (0, -5)
11) x-int: (-5 , 0) ; y-int: (0, -3)
12) x-int: (4, 0) ; y-int: (0, -12)
13) x-int: (0, 0) ; y-int: (0, 0)
14) x-int: (5, 0) ; y-int: none

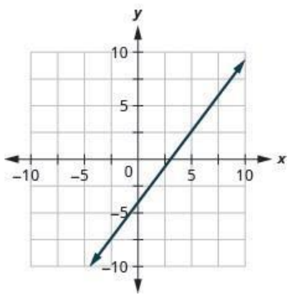
2.3 Slope-Intercept Form

1) Slope: -2/3; y-int: (0, 4)
2) Slope: -1; y-int: (0, -5)
3) Slope: -3/5; y-int: (0, 1)
4) Slope: 53; y-int: (0, -6)
5) Slope: 4/5; y-int: (0, -8/5)
6) Slope: -4; y-int: (0, 9)
7) y = 2x + 5
8) y = x – 4
9) y = -3x – 1
10) y = 13x + 1

11)



13)



2.4 Point-Slope Form

1) $y + 4 = \frac{3}{5}(x - 3)$
2) $y - 4 = -\frac{5}{4}(x + 1)$
3) $y - 4 = -2(x - 2)$
4) $y = 3$
5) x = -6
6) $y - 6 = -(x - 3)$
7) $y - 1 = \frac{5}{2}(x - 3)$
8) $y - 4 = -\frac{2}{5}(x - 1)$
9) $x = -5$
10) y = 3

2.5 Parallel & Perpendicular Lines

1) m = 2
2) m = -23
3) m = 4
4) m = -103
5) m = 65
6) m = -34
7) m = 0
8) m = 3
9) m = -3
10) m = 2
11) m = -38
12) m = 13
13) Neither
14) Neither
15) Parallel
16) y = -2x +5
17) y = 35x+5
18) y = -4x-3
19) y = -2
20) y = x-1
21) y = 2x-11
22) x = 5
23) y = -2x+5

4.1 Solving Systems of Equations by Graphing

1) Yes
2) No
3) No
4) (-2, -3)
5) (2, -2)
6) (1, -1)
7) (-1, -4)
8) Infinite number of solutions
9) No solution

4.2 Solving Systems by Substitution

1) (-2, 0)
2) (0, 2)
3) (10, -1)
4) (3, 2)
5) Infinite number of solutions
   (x, -x/3 + 4/3)
6) No solution

4.3 Solving Systems by Addition

1) (1, 2)
2) (-5, 1)
3) (-2, -3)
4) (0, 2)
5) (3, -6)
6) (-6, 10)
7) 15 & 24
8) -25 & -10

4.5 Application: Value Problems

1) 347 child tickets & 206 adult tickets are sold
2) 13 dimes and 29 quarters
3) He should invest $32,800 in stock & $7,200 in bonds.
4) $10 bills = 27 & $20 bills = 3
5) $12,000 should be invested at 5.25% & $13,000 should be invested at 4%.

4.6 Application: Mixture Problems

1) 16 pounds of nuts & 4 pounds of chocolate chips
2) 3 pounds of peanuts & 2 pounds of cashews
3) 80% = 28 liters & 30% = 42 liters
4) 12% = 50 liters & 25% = 15 liters
5) 70% = 80 liters & 40% = 160 liters

5.1 Exponent Properties

1) a^6
2) $8a^3b^3$
3) 3. $-18x^{11}$
4) 2^{14}
5) $-6x^5$
6) $4x^5$
7) x^2
8) -6c
9) $3a^2b^3$
10) $6cd^3$
11) $11y^7$
12) $125x^6y^{12}$
13) $216x^{13}y^{20}$
14) $49x^8y^5$
15) $64x^9y^9$
16) x^9
17) x^{14}
18) $-32x^{20}$
19) $18x^{10}$
20) $27s^3t^{36}$
21) $243m^5n^{30}$
22) $\frac{x^{28}}{y^{12}}$
23) x^5y^{15}
24) 3^34^9
25) $-9x^2$

5.2 Negative Exponents

1) -1
2) 1
3) -1
4) 1
5) 0
6) 2
7) 0
8) z^4
9) $\frac{b^3}{a^7}$
10) $-\frac{8}{x^6}$
11) $\frac{1}{25}$
12) $\frac{5}{6}$
13) a^4
14) $-\frac{1}{64}$
15) x^6

5.3 Scientific Notation

1) 5.75×10^2
2) 8.74×10^4
3) 6.43×10^{-1}
4) 8.02×10^{-4}
5) 25.4
6) 6190
7) .464
8) 0.007
9) n=13
10) 3.09936×10^{16} ft
11) 3.55×10^5
12) 1.4×10^{-3}
13) 1.56×10^{-3}
14) 18.148148×10^3
15) 55.405405×10^{-6}

5.4 Introduction to Polynomials

1) (a) 52 (b) 40 (c) 4
2) (a) monomial, degree 0
   (b) polynomial, degree 3
   (c) trinomial, degree 4
   (d) binomial, degree 2
   (e) monomial, degree 10
3) (a) $40y^2$
   (b) $22pq^3$
4) $11y^2 - 10y + 2$
5) (a) $10m^2 + 3mn - 8n^2$
   (b) $3b^2 - 3ab$
   (c) $5p^3 - 6p^2q + pq^2$
6) $2m^2 - 7m + 4$
7) $3x^2 – x + 4$
8) $-8z + 2$

5.5 Multiply Polynomials

1) $-6p + 42$
2) $32k^2 + 16k$
3) $-18n^3 - 21n^2$
4) $n^2 + 14n + 48$
5) $b^2 - 2b - 15$
6) $4r^2 - 24r - 64$
7) $49n^2 - 36$
8) $25x^2 - 15xy + 2y^2$
9) $6r^3 - 43r^2 + 12r - 35$
10) $12n^3 - 20n^2 + 38n - 20$
11) $18x^2 - 15x - 12$
12) $-7x^2 + 49x - 70$
13) $x^3 - 3x^2 - 10x + 21$
14) $10x + 2$
15) $P = 4w + 4$; $A = w^2 + 2w$
16) $P = 6w - 6$; $A = 2w^2 - 3w$

5.6 Muliply Special Products

18) $x^2 + 10x + 25$
19) $4x^2 - 4x + 1$
20) $9x^2 - 12xy + 4y^2$
21) $x^2 - 6xy + 9y^2$
22) $x^2 + x + 1/4$
23) $3a^2 - 24a + 48$
24) $-5w^2 + 10wy - 5y^2$
25) $x^2 - 9$
26) $d^2 - 49$
27) $9x^2 - 1$
28) $25x^2 - 4y^2$
29) $c^2 - 121$
30) $x^2 - \frac{1}{4}$
31) $5x^2 - 5$
32) $-12x^3 + 13x$
33) $x^3 - 15x^2 + 75x - 125$
34) $8x^3 - 36x^2 + 54x - 27$

5.7 Divide Polynomials

1) $\frac{2m^3}{3n^5}$
2) $-\frac{3y^8}{4x^3}$
3) $\frac{2b^4}{a^4}$
4) $3n^3 + 2n^2$
5) $2a - 1 - \frac{4}{5n}$
6) $6x - 10y - 4x^2y$
7) $y + 4$
8) $a - 7$
9) $4x + 3$
10) $p + 3 - \frac{8}{(p+8)}$
11) $3b - 2 + \frac{6}{(b+1)}$
12) $16y^2 + 12y + 9$
13) (a^2-3)

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